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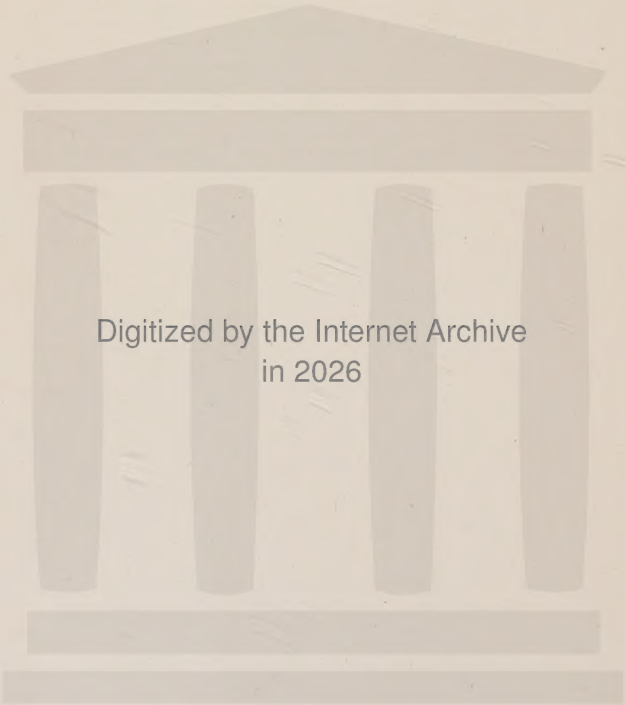

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WENTWORTH-SMITH MATHEMATICAL SERIES

JUNIOR HIGH SCHOOL MATHEMATICS

BOOK I

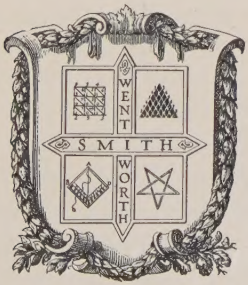
BY

GEORGE WENTWORTH

DAVID EUGENE SMITH

AND

JOSEPH CLIFTON BROWN



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PREFACE

A proper curriculum for junior high schools and six-year high schools demands, in the opinion of many teachers, a course in mathematics which introduces concrete, intuitional geometry and the simple uses of algebra in the lower classes. This book is intended to meet such a demand for the lowest class.

Arithmetic furnishes the material for the first half of the book. The second half of the book is devoted to intuitional and constructive geometry, a subject which is more concrete than algebra and which admits of more simple illustration. Both arithmetic and geometry are arranged with respect to large topics, the intention being to avoid the lack of system which so often deprives the student of that feeling of mastery which is his right and his privilege. These large topics are set forth clearly in the table of contents.

In this book there is gradually introduced the algebraic formula, so that the student is aware of the value of algebra as a working tool before he proceeds to the study of Book II. The text is so arranged that with entire convenience to the teacher and student the work in arithmetic may be taken parallel with the work in geometry, or the work in geometry may precede the work in arithmetic.

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Any remodeling of the elementary curriculum that sacrifices thorough training in arithmetic would be a transitory thing, and any anæmic course in mathematics that leaves a student too languid intellectually to pursue the subject further with success is foredoomed to failure. This book gives to arithmetic the place due to its fundamental importance, and it adheres to a sane and usable topical plan throughout the development

of the various subjects treated. Because of this the authors believe that they have here produced a textbook suited to the needs of a rapidly growing class of schools and that they have not failed in any respect to adhere to the best standards of mathematics and pedagogy. As to material for daily drill, teachers should consult page 105.

Book II is devoted to algebra and arithmetic, each making use of the important facts presented in Book I and each including those large and important topics which are valuable in the elementary education of every boy and girl. The two books thus work together to a common purpose, the first being the more concrete and preparing by careful steps for the second, and the second blending with the first in presenting to the student a well-organized foundation for the more formal treatment of the mathematics which naturally follows.

The authors take great pleasure in being able to include in this work a number of decorative illustrations of early mathematical instruments and their uses, by Mr. T. M. Cleland. They feel sure that teachers and students will welcome this innovation in the preparation of textbooks in mathematics, and will appreciate such a combination of the work of the artist with that of the mathematician.

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JUNIOR HIGH SCHOOL MATHEMATICS

BOOK I

PART I. ARITHMETIC

I. ARITHMETIC OF THE HOME

Nature of the Work. You have already completed the arithmetic which treats of ordinary computation, and have probably learned how to use the common tables of measures and how to find a given per cent of a number. You are therefore now ready to consider the most important applications of arithmetic — those which relate to the home, the store, the farm, the most common industries, and the bank.

In taking up these applications of arithmetic we shall review the operations with numbers and shall pay particular attention to those short cuts in computation that the business man needs to know and that are useful in various kinds of work. We shall also take up again the important subject of percentage, which enters into every kind of business, and shall treat of it from the beginning.

In this book notes in this type are chiefly for the teacher's use.

The teacher is advised to spend a little time in discussing this page, in order to take stock of what has already been studied and to set forth the general nature of the work which is ahead of the students. The reading of a few interesting problems which the students will meet adds a motive to the work.

Cash Account. It is important that every boy and girl should early in life form the habit of keeping a personal cash account. It is customary to write the receipts on the left side of the account (called the *debit* side) and to write the payments on the right side (called the *credit* side), thus:

	RECEIPTS					PAYMENTS					
	1920					1920					
a	Jan.	6	Cash on hand	2	75	Jan.	7	Paper		05	*g*
b		7	From uncle		50		8	Marbles		25	*h*
c		8	From father		20		8	Book		50	*i*
d		9	Errands		35		10	Balance	3	00	*j*
e				3	80				3	80	*k*
f	Jan.	10	Balance	3	00						

Here *a* shows cash on hand when this page of the account is begun; *b*, *c*, and *d* are receipts; *e* is the sum of these items on the day the account is *balanced*. On the right side *g*, *h*, and *i* are amounts paid. To find the amount on hand we subtract 80¢, the sum of 5¢, 25¢, and 50¢, from \$3.80, and find that the *balance* is \$3.00, which we write at *j* and again at *f*. To check the work, *g*, *h*, *i*, and *j* are added, and the sums, at *e* and *k*, must agree. In keeping a cash account the symbol \$ is usually omitted.

Check. To check the work in addition always add a column from the top down after having added it from the bottom up. *Every computation should be checked.*

One of the first things one must learn in business is the necessity of checking every computation. Teachers should impress this important business rule upon the students. When the other operations are met in this work, the teacher should explain the proper checks if necessary, but these checks should be known to the students from the work of previous years.

Exercise 1. Cash Accounts

Given the following items, make out the cash accounts and balance them:

1. Receipts: Oct. 25, cash on hand, 45¢; Oct. 26, earned by errands, 26¢; Oct. 27, earned by errands, 35¢. Payments: Oct. 26, ball, 20¢; Oct. 28, bat, 25¢. Balanced Oct. 29.

2. Receipts: Sept. 7, cash on hand, $3.20; Sept. 8, earned by errands, 40¢; Sept. 9, gift from father, 50¢; Sept. 10, earned by cleaning automobile, 30¢. Payments: Sept. 9, football pants, $1.75; Sept. 10, share in football, 50¢; Sept. 11, book, 30¢. Balanced Sept. 12.

3. Receipts: Nov. 5, cash on hand, 75¢; Nov. 5, earned by caring for furnace, 20¢; Nov. 6, earned by cleaning automobile, 25¢; Nov. 7, earned by caring for furnace, 15¢. Payments: Nov. 7, cap, 75¢. Balanced Nov. 8.

4. Receipts: May 1, cash on hand, $38.75; May 3, J. C. Williams, $20; May 5, R. S. James, $36.85. Payments: May 4, groceries, $8.75; meat, $2.80. Balanced May 6.

5. Receipts: May 9, cash on hand, $275.25; May 10, R. J. Benjamin, $73.75; May 12, S. K. Henry, $250.75. Payments: May 9, rent, $85; May 12, clerks, $75; May 15, coal, $22.50; May 17, account book, $1.50; May 22, telephone, $2.75; May 23, gas, $5.60. Balanced May 23.

6. Receipts: Aug. 1, cash on hand, $178.50; Aug. 2, P. M. Myers, $87.60; Aug. 3, A. B. Noyes, $49.75; Aug. 5, R. L. Dow, $78.80. Payments: Aug. 2, rent, $50; Aug. 3, electric light, $9.75; Aug. 4, gas, $4.80; Aug. 5, J. P. Sinclair, wages, $16.75; Aug. 5, telephone, $2.50; Aug. 6, M. R. Roe, wages, $15. Balanced Aug. 7.

Problems of this kind should be made up by the students, who should be urged to keep their personal accounts.

Household Account. The following will be found a convenient form for keeping an account of household expenses:

1920			Receipts		Payments	
Sept.	1	Cash on hand	73	92		
		Rent			12	
		Gas			2	20
		Electric light			1	40
	8	Groceries, 1 wk.			4	90
		Milk, 1 wk.			1	26
		Butter, eggs			2	35
		Ice				35
		Carfare				30
		Laundry				40
	10	Church			1	
		Charity			2	50
	11	Concert				75
		Gloves			1	50
		Help, 2 da.			3	
	16	Cash rec'd	12			
		Balance			46	01
			85	92	85	92
	16	Cash on hand	46	01		

The balance should agree with the cash on hand. The balance is found by adding the items of payments and subtracting this sum from the sum of the receipts. In this case we have $\$85.92 - \$39.91$. The sum of the receipts will then agree with the sum of the payments and the balance. Bookkeepers usually write the balance in red.

Exercise 2. Household Accounts

1. Extend the account on page 4 from Sept. 16 to Sept. 24 by including the following receipts and payments: Receipts: Sept. 18, Mrs. Adams, loan repaid, $3.50; household allowance, $20. Payments: Sept. 18, telephone, $1.75; vegetables, 65¢; Sept. 19, trolley tickets, $1; shoes, $4; ribbon, 75¢; Sept. 21, umbrella, $2.50; belt, 75¢; Sept. 23, laundry, 90¢; charity, $1.50; groceries, $5.15; ice, 45¢.

Make out and balance the following accounts:

2. Receipts: Oct. 9, cash on hand, $15.42; Oct. 10, allowance, $15; Oct. 12, loan repaid by Mr. Green, $5.80. Payments: Oct. 10, insurance, $2.50; help, $1.50; lecture, 50¢; Oct. 11, dress goods, $5.80; trimming, $1.10; Oct. 12, raincoat, $6.50; shoes repaired, $1.25; Oct. 13, groceries, $3.90; milk, $2.80; Oct. 15, carfare, 30¢; flowers, 40¢; dressmaker, 3 da. @ $1.80. Balanced Oct. 16.

3. Receipts: Nov. 18, cash on hand, $12.67; Nov. 19, allowance, $22.50. Payments: Nov. 19, cleaning gloves, 25¢; dress goods, $9.60; Nov. 20, telephone, $1.90; removing garbage, 75¢; Nov. 21, gas, $2.30; electric light, $1.80; magazines, 30¢; Nov. 22, books, $2.60; groceries, $7.30; meat, $1.20; charity, $1.25. Balanced Nov. 23.

4. Write an account setting forth the reasonable expenses of a week for a family of two adults and three children, to include groceries, meat, milk, gas, electric light, telephone, and such other items as would be found in an account of such a family in your vicinity. Take the balance on hand as $4.80 and the weekly allowance for household expenses, exclusive of rent and clothes, as $16.

For Ex. 4 the students should be asked to make inquiry at home as to prices and reasonable purchases.

Need for knowing about Per Cents. John Adams is manager of the baseball team of a junior high school. He finds that he can buy 4 bats at 50¢ each. The next day he sees that they have been marked down 10 per cent. Hence he should know what *per cent* means.

It is probable that everyone in the class knows. In that case some student may tell the meaning; otherwise the subject may be discussed after page 7 has been read. This page is merely a preparatory reading lesson for those who may not already have studied percentage, which subject will now be considered.

John's mother wishes to buy a dress for his sister. The price was $12, but the dress has been marked down 15 per cent. If she wishes to buy the dress, she should know what this means. Do you know what it means?

The teacher has to make a report of the number of students tardy or absent last week. She says in the report that 4 per cent of the students were tardy and 3 per cent were absent. Do you know what this means?

There were 10 questions on an examination in arithmetic and a boy answered 90 per cent of them correctly. Do you know how many questions he answered correctly? Do you know how many he did not answer correctly?

A man wishes to buy an automobile. He can buy a new one of the kind he likes for $1200, but one of the same make that is nearly new is offered for 30 per cent less than this price. Do you know how to find what the man would have to pay for the second-hand car?

Do you know the meaning of the symbols 10%, 15%, 4%, 3%, 90%, and 30%?

It is not necessary that any student should be able to answer these questions. What is of importance is that each member of the class should see that per cents are frequently encountered and that everyone must know what they mean.

Per Cent. Another name for "hundredths" is *per cent.* For example, instead of saying "ten hundredths" we may say "ten per cent." The two expressions mean the same.

If John Adams finds that some bats which are marked \$2 can be bought for 10 per cent less than the marked price, this means that they can be bought for $\frac{10}{100}$, or $\frac{1}{10}$, less than \$2; that is, they can be bought for \$2 less $\frac{1}{10}$ of \$2, or \$2 – \$0.20, or \$1.80.

Symbol for Per Cent. There is a special symbol for per cent, %. Thus we write 20% for 20 per cent, or 0.20.

Hundredths written as Per Cents. Because hundredths and per cents are the same, any common fraction with 100 for its denominator may easily be written as per cent. Thus

$$\frac{5}{100} = 5\% \qquad \frac{1}{100} = 1\% \qquad \frac{75}{100} = 75\% \qquad \frac{135}{100} = 135\%$$

Exercise 3. Reading Per Cents

All work oral

Read the following as per cents:

1. $\frac{3}{100}$.	**4.** $\frac{10}{100}$.	**7.** $\frac{20}{100}$.	**10.** $\frac{65}{100}$.	**13.** $\frac{85}{100}$.
2. $\frac{6}{100}$.	**5.** $\frac{12}{100}$.	**8.** $\frac{35}{100}$.	**11.** $\frac{70}{100}$.	**14.** $\frac{96}{100}$.
3. $\frac{8}{100}$.	**6.** $\frac{15}{100}$.	**9.** $\frac{50}{100}$.	**12.** $\frac{80}{100}$.	**15.** $\frac{100}{100}$.

Read the following as hundredths:

16. 6%.	**18.** 22%.	**20.** 43%.	**22.** 66%.	**24.** 100%.
17. 9%.	**19.** 37%.	**21.** 50%.	**23.** 72%.	**25.** 300%.

26. Read $\frac{250}{100}$ as per cent, and 125% as hundredths.

27. Read as per cents: $\frac{6\frac{1}{4}}{100}$, $\frac{16\frac{2}{3}}{100}$, $\frac{\frac{1}{8}}{100}$, $\frac{37\frac{1}{2}}{100}$, $\frac{\frac{3}{8}}{100}$, $\frac{87\frac{1}{2}}{100}$.

Important Per Cents. Mr. Fuller says that he sold a used car for 50% of what it cost him. He sold the car for how many hundredths of what it cost him? Express the answer as a common fraction in lowest terms.

If a baseball team played 36 games and lost 25% of them, how many hundredths of the games did it lose? Express the answer as a common fraction in lowest terms.

From these two examples, and by dividing 25% by 2, we see that

$$50\% = \tfrac{1}{2} \qquad 25\% = \tfrac{1}{4} \qquad 12\tfrac{1}{2}\% = \tfrac{1}{8}$$

If this circle is called 100, how much is the shaded part? What per cent of the circle is the shaded part?

Read $0.33\frac{1}{3}$, using the words "per cent"; using the word "hundredths." How many thirds are there in 1? How many times is $33\frac{1}{3}\%$ contained in 1? Then $33\frac{1}{3}\%$ is what part of 1?

From these two examples, and by taking $2 \times 0.33\frac{1}{3}$ and $\frac{1}{2}$ of $0.33\frac{1}{3}$, we see that

$$33\tfrac{1}{3}\% = \tfrac{1}{3} \qquad 66\tfrac{2}{3}\% = \tfrac{2}{3} \qquad 16\tfrac{2}{3}\% = \tfrac{1}{6}$$

Read 0.2 and 0.20, using the word "hundredths" in each case; using the words "per cent." What relation do you see between 0.2 and 0.20? between 0.2 and 20%? between $\frac{1}{5}$ and 20%?

What is the relation of $\frac{4}{10}$ to $\frac{40}{100}$? to 40%?

What is the relation of 0.60 to $\frac{3}{5}$? of 0.80 to $\frac{4}{5}$?

From answers to these questions we see that

$$20\% = \tfrac{1}{5} \qquad 40\% = \tfrac{2}{5} \qquad 60\% = \tfrac{3}{5} \qquad 80\% = \tfrac{4}{5}$$

In the same way it is easily seen that

$$75\% = \tfrac{3}{4} \qquad 37\tfrac{1}{2}\% = \tfrac{3}{8} \qquad 62\tfrac{1}{2}\% = \tfrac{5}{8} \qquad 87\tfrac{1}{2}\% = \tfrac{7}{8}$$

Such per cent equivalents should be drilled upon so thoroughly that the mention of one form automatically suggests the other.

Exercise 4. Finding Per Cents

All work oral

1. A woman who had set aside \$16 for household expenses for a week finds that she has spent 50% of it. How much money has she spent?

Find 50% of each of the following numbers:

2. 18. 3. 20. 4. 36. 5. 64. 6. 400.

7. If this square represents a box cover which contains 100 sq. in., how many square inches are shaded? How many fourths of the cover are shaded? How many hundredths of the cover are shaded? What per cent of the cover is shaded?

8. A man with an income of \$4 a day spends 25% of it for food. How much money does he spend for food?

Find 25% of each of the following numbers:

9. 16. 10. 36. 11. 48. 12. 5. 13. 240.

Find the values of the following:

14. 20% of 75. 17. 80% of 45. 20. 75% of 800.
15. 40% of 25. 18. $33\frac{1}{3}$% of 75. 21. $87\frac{1}{2}$% of 1600.
16. 60% of 35. 19. $16\frac{2}{3}$% of 66. 22. $66\frac{2}{3}$% of 6000.

23. If you answer correctly $66\frac{2}{3}$% of the questions on an examination paper and there are 12 questions in all, how many questions do you answer correctly?

24. If a boy is at bat 10 times and makes base hits 20% of the times, how many base hits does he make?

25. If a merchant gains $33\frac{1}{3}$% on \$1500, what fractional part of \$1500 does he gain? How much does he gain?

Per Cents and Common Fractions. We have learned that

$$87\tfrac{1}{2}\% = \frac{87\frac{1}{2}}{100} = \frac{175}{200} = \frac{7}{8}.$$

To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and then reduce to lowest terms.

It is sometimes convenient to use one form, and sometimes another. Thus, if we are multiplying by 26.9%, it is easier to think of the multiplier as 0.269; but if we are multiplying by $66\frac{2}{3}\%$, it is easier to think of it as $\frac{2}{3}$.

We know that $\frac{2}{3}$ may be reduced to hundredths by multiplying each term by $33\frac{1}{3}$. We then have

$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} = 66\tfrac{2}{3}\%.$$

To express a common fraction as per cent, reduce it to a common fraction with 100 for the denominator, and then write the numerator followed by the symbol for per cent.

We may also reduce $\frac{2}{3}$ to hundredths by dividing 2 by 3, thus:

$$\tfrac{2}{3} = 2 \div 3 = 2.00 \div 3 = 0.66\tfrac{2}{3} = 66\tfrac{2}{3}\%.$$

Exercise 5. Per Cents and Common Fractions

Express as common fractions in lowest terms:

1. 12%. **3.** 36%. **5.** 64%. **7.** 29%. **9.** $17\frac{1}{2}\%$.

2. 24%. **4.** $3\frac{1}{8}\%$. **6.** 32%. **8.** 45%. **10.** $66.6\frac{2}{3}\%$.

Express the following as per cents:

11. $\frac{3}{4}$. **13.** $\frac{5}{8}$. **15.** $\frac{4}{5}$. **17.** $\frac{3}{2}$. **19.** $\frac{1}{16}$. **21.** $\frac{7}{20}$.

12. $\frac{3}{8}$. **14.** $\frac{3}{5}$. **16.** $\frac{5}{5}$. **18.** $\frac{7}{8}$. **20.** $\frac{3}{16}$. **22.** $\frac{8}{25}$.

Per Cents as Decimals. Since 25.5% and 0.255 have the same value, we see the truth of the following:

To express as a decimal a number written with the per cent sign, omit the sign and move the decimal point two places to the left, prefixing zeros if necessary.

When we omit the per cent sign we must indicate the hundredths in some other way, as by moving the point two places to the left.

Thus $2\frac{1}{4}\% = 0.02\frac{1}{4}$, or 0.0225; $325\% = 3.25$; $0.7\% = 0.007$.

Exercise 6. Per Cents as Decimals

Examples 1 to 13, oral

1. If 3% of the students of this school are absent to-day how many are absent out of every 100? out of every 200?

2. How much is $\frac{1}{100}$ of \$400? 1% of \$400?

3. How much is 0.05 of \$100? 5% of \$100? 5% of \$200? 5% of \$2000? 5% of \$4000?

Read as decimals, whole numbers, or mixed decimals:

4. 35%. **6.** 27%. **8.** 325%. **10.** 365%. **12.** 400%.

5. 72%. **7.** 42%. **9.** 225%. **11.** 425%. **13.** 200%.

14. Express $\frac{3}{4}\%$ as a decimal; as a common fraction.

15. Express $\frac{5}{8}\%$ as a decimal; as a common fraction.

16. Express $666\frac{2}{3}\%$ as a decimal; as an improper fraction.

17. How much is 0.35 of \$300? 35% of \$300?

18. How much is $2\frac{3}{4} \times \$650$? $2.75 \times \$650$? 275% of \$650? $3\frac{1}{4} \times \$750$? $3.25 \times \$750$? 325% of \$750?

Express as decimals, whole numbers, or mixed decimals:

19. 24%. **21.** $33\frac{1}{3}\%$. **23.** 300%. **25.** 3000%.

20. 36%. **22.** 0.8%. **24.** 250%. **26.** 0.008%.

Decimals as Per Cents. Since per cent means hundredths, to express a decimal as per cent we have to consider only how many hundredths the decimal represents.

1. Express 0.5 as per cent.

Since $0.5 = 0.50$, we see that 0.5 is the same as 50 hundredths, or 50%.

2. Express 0.625 as per cent.

Since $0.625 = 62.5$ hundredths, or $62\frac{1}{2}$ hundredths, we see that 0.625 is the same as 62.5%, or $62\frac{1}{2}$%.

3. Express 0.00375 as per cent.

Since $0.00375 = 0.00\frac{3}{1}\frac{7}{0}\frac{5}{0} = 0.00\frac{3}{8}$, we see that $0.00375 = \frac{3}{8}$%.

4. Express $4.2\frac{1}{2}$ as per cent.

Since $4.2\frac{1}{2} = 4\frac{2}{1}\frac{5}{0}$, we see that $4.2\frac{1}{2} = 425$%.

Therefore, *to express a decimal as per cent, write the per cent sign after the number of hundredths.*

Exercise 7. Decimals as Per Cents

Examples 1 to 6, oral

1. A foot, being $0.33\frac{1}{3}$ yd., is what per cent of a yard?

2. A peck, being 0.25 bu., is what per cent of a bushel?

3. A quart, being 0.125 pk., is what per cent of a peck?

4. Express 0.24 mi. as per cent of a mile.

5. Express 1 oz. as per cent of 10 oz.; of 100 oz.

6. Express 0.2 ft. as per cent of 1 ft.; of 2 ft.

Express the following as per cents:

7. 0.7. **9.** 0.42. **11.** 0.625. **13.** $6.66\frac{2}{3}$.

8. 0.8. **10.** 0.39. **12.** 0.375. **14.** 0.00875.

Finding Per Cents. We have now found that certain important fractions like $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{5}$ are easily expressed as per cents; that any fraction can be expressed as a per cent by first reducing it to hundredths; and that per cents can easily be expressed as common fractions.

If we wish to find 50%, 25%, $12\frac{1}{2}$%, $33\frac{1}{3}$%, $16\frac{2}{3}$%, or 20% of a number, it is easier to use the equivalent common fraction, as in the first of these problems:

1. If Kate wishes to buy a suit that is marked \$12, and finds that it is to be marked down $12\frac{1}{2}$%, how much will the suit cost her after it is marked down?

Since $12\frac{1}{2}\% = \frac{1}{8}$, we need simply to take $\frac{1}{8}$ of \$12, and this is the amount the suit is to be marked down.

Then we have $\frac{1}{8}$ of \$12 = \$1.50;

and \$12 − \$1.50 = \$10.50, cost.

2. Robert wishes a Boy Scout suit. The marked price of the suit he wants is \$7.50, but he finds that to-morrow at a bargain sale this suit is to be marked down 15%. How much will it then cost him?

Since 15% = 0.15 we should find 0.15 of \$7.50. We then have

0.15 of \$7.50 = \$1.125,

and \$7.50 − \$1.13 = \$6.37, cost.

\$7.50	\$7.50
0.15	1.12
3750	\$6.38, cost
750	
\$1.1250	

The dealer will probably deduct only \$1.12, neglecting the 5, in order to make the computation easier and to take advantage of the half cent himself. There is no general custom as to using a fraction of a cent. The student should consider $\frac{1}{2}$¢ or more as a whole cent in each operation except in cases of discount.

To find a given per cent of a number, express the per cent as a common fraction or as a decimal and multiply the number by this result.

Exercise 8. Finding Per Cents

Examples 1 to 20, oral

1. A man spends 20% of his income for rent. What fraction of his income does he spend for rent?

2. A man saves 10% of his income. His income is \$90 a month. How much does he save in a month?

3. A girl spends for clothes 25% of her allowance. Express this per cent as a common fraction.

Express as common fractions or as whole or mixed numbers:

4. 7%. 5. 30%. 6. 40%. 7. 75%. 8. 60%. 9. 90%. 10. 1%. 11. 2%. 12. 4%. 13. 100%. 14. 200%. 15. 500%. 16. 125%. 17. 250%. 18. 375%.

19. How much is 10% of 420 lb.? of \$250? of 30 yd.?

20. Milk yields in butter about 4% of its weight. How much butter will 25 lb. of milk yield?

21. If a merchant pays $1\frac{1}{4}$¢ apiece for pencils and sells them at a profit of 50% on the cost price, what is the selling price per hundred?

22. If a butcher buys a certain kind of meat at 15¢ a pound and sells it at a profit of 20% on the cost price, at what price does he sell it per pound?

23. If a dealer pays $7\frac{1}{2}$¢ a quart for milk and sells it at a profit of 20% on the cost price, how much does he receive for 200 qt. of milk?

Find the values of the following:

24. 30% of 2400.
25. 16% of \$3600.
26. 35% of \$825.
27. 75% of \$6280.
28. 27% of 3700 ft.
29. $3\frac{1}{3}$% of \$4275.

Exercise 9. Reading the Gas Meter

1. Here is a picture of the three dials on a gas meter. The left-hand dial indicates ten thousands, the middle dial indicates thousands, and the right-hand dial indicates

hundreds. The dial shows that 64,300 cu. ft. of gas has passed through the meter. If this is the reading for May 1, and the reading for April 1 was 62,300, how much was the gas bill for April at 80¢ per M (1000 cu. ft.)?

In the above case more than 64,300 cu. ft. has passed through the meter, but we read only to the hundred last passed. Notice that the middle dial is read in the opposite direction from the others.

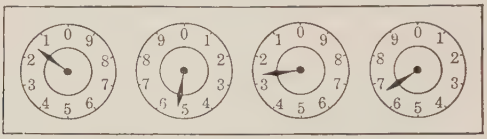
2. Read this meter. If the reading was 52,700 a month ago, how much is the gas bill for the month at \$1 per M?

3. If the gas consumed by a family in January was 3200 cu. ft., and that consumed in June was 10% less than in January, how much was the gas bill for each of these months at \$1.25 per M?

4. A certain gas company deducts 10% from bills paid before the tenth of the month. If the readings are 72,400 and 74,200 on the first days of two consecutive months, and the rate for gas is \$1.30 per M, how much will be saved by paying the bill before the tenth of the month?

Exercise 10. Reading the Electric Meter

1. Mr. Jacobs lights his store by electricity. The electricity is measured in kilowatt hours (K.W.H.), and the meter shows thousands, hundreds, tens, and units. He reads the meter in a way similar to that in which he reads the gas meter. Notice that the second and fourth dials are read clockwise and that the first and third are read counterclockwise. Read the meter shown:



The technical meaning of the K.W.H. in the science of electricity need not be considered in the mathematics of the junior high school.

2. If Mr. Jacobs in June uses 40 K.W.H. at 15¢ and 5 K.W.H. at 8¢, how much is his bill for the month?

Companies charge large users differently according to the number of lights or machines. For example, in most places 10 lights would cost more in proportion than 200 lights.

3. If Mr. Jacobs in the month of March uses in his house 18 K.W.H. at 14¢ and has a reduction of 4% on the bill if paid before the 15th of April, how much is his bill if he takes advantage of the reduction?

Find the amount of each of the following bills:

4. 12 K.W.H. at 15¢, 3 K.W.H. at 7¢, less $\frac{1}{2}$¢ per K.W.H. on account of prompt payment.

5. 28 K.W.H. at 16¢, 6 K.W.H. at 10¢, less $\frac{1}{2}$¢ per K.W.H.

The student should actually read the electric meter in the school or at his home if this can be done conveniently.

Three Important Problems of Percentage. There are three important problems of percentage. The first, which is by far the most important one, is that of finding a given per cent of a number. This has already been considered. The second problem of importance is to find what per cent one number is of another, and this will be considered on page 18. The third problem of importance is to find the number of which a given number is a given per cent, and this will be considered on page 21.

The problems on pages 18 and 21 depend upon the following principle:

Given the product of two factors and one of the factors, the other factor may be found by dividing the product by the given factor.

That is, if we have given 10, the product of 2 and 5, we can find the factor 2 by dividing the given product by 5 and the factor 5 by dividing the product by 2.

In this review of percentage we shall confine the applied problems largely to those relating to home interests, but shall occasionally introduce other problems for the purpose of variety.

Exercise 11. Product and One Factor

All work oral

The first number in each of the following examples being the product of two factors, and the second number being one of these factors, find the other factor:

1. 72, 8.	**4.** 36, 9.	**7.** 90, 9.	**10.** 300, 10.
2. 72, 9.	**5.** 80, 10.	**8.** 60, 6.	**11.** 9, 3.
3. 36, 4.	**6.** 80, 8.	**9.** 300, 30.	**12.** $\frac{9}{10}$, 3.

13. I am thinking of the number which, multiplied by 9, gives the product 63. What is the number?

Finding what Per Cent One Number is of Another. The second problem of importance in percentage mentioned on page 17 is to find what per cent one number is of another.

For example, if in a certain test Anna solved correctly 8 problems out of 12, what per cent of the problems did she solve correctly?

Here we have a certain per cent $\times 12 = 8$; that is, we have the product (8) and one factor (12), to find the other factor (a certain per cent). Therefore

$8 \div 12 = 0.66\frac{2}{3} = 66\frac{2}{3}\%$, the per cent solved correctly.

Exercise 12. Product and One Factor

All work oral

1. If there are 30 students enrolled in a class and 3 of them are absent to-day, what per cent are absent?

2. On an automobile trip of 60 mi., what per cent of the distance has a man made when he has traveled 30 mi.?

3. During a series of games Fred was at bat 36 times and made 9 base hits. What was his batting average?

4. If a baseball team wins 14 games out of 20 games played, what per cent of the games does it win?

Find what per cent the second number is of the first:

5. 4, 2.	9. 6, 2.	13. 45, 15.	17. 64, 8.
6. 4, 3.	10. 40, 30.	14. 75, 15.	18. 64, 16.
7. 4, 4.	11. 30, 15.	15. 68, 34.	19. 36, 18.
8. 5, $2\frac{1}{2}$.	12. 90, 45.	16. 72, 36.	20. 15, $7\frac{1}{2}$.

21. If a book has 240 pages, what per cent of the number of pages have you read when you have read through page 24? when you have read through page 48? when you have read through page 72? through page 120?

Application to Written Exercises. You now understand the second important problem in percentage. We shall consider once more the first problem, and then the application of the second to written exercises.

1. A man bought an automobile for \$800 and sold it at a profit of 25% on the cost. How much did he gain?

Here we have to find 25% of \$800, or 0.25 of \$800; that is, we have two factors given, 0.25 and \$800, to find the product. Therefore we have

$$0.25 \times \$800 = \$200.$$

That is, the man gained \$200.

2. A man bought an automobile for \$800 and sold it at a profit of \$200. What per cent of the cost did he gain?

Here we have \$200 equal to some per cent of \$800; that is, we have given the product (\$200) of two factors and one of the factors (\$800), to find the other factor. Therefore we have

$$\$200 \div \$800 = 0.25 = 25\%.$$

That is, the man gained 25% of the cost.

Exercise 13. Product and One Factor

1. \$260 is what per cent of \$5200? of \$7800?

2. \$22.50 is what per cent of \$450? of \$67.50?

3. \$20.24 is what per cent of \$506? of \$404.80?

4. \$58.20 is what per cent of \$931.20? of \$349.20?

5. A foot is what per cent of a yard? of 2 yd.? of 8 yd.?

6. A quart is what per cent of a gallon? of 16 gal.?

7. $\frac{3}{4}$ is what per cent of $\frac{5}{8}$? $\frac{5}{8}$ is what per cent of $\frac{3}{4}$?

8. 35% is what per cent of 70%? of 140%? of 210%?

9. $33\frac{1}{3}\%$ is what per cent of $66\frac{2}{3}\%$? of 1? of $133\frac{1}{3}\%$?

10. $66\frac{2}{3}\%$ is what per cent of $33\frac{1}{3}\%$? of $66\frac{2}{3}\%$? of 2?

11. If an employer reduces the working day for his men from 9 hr. to 8 hr., what is the per cent of reduction?

12. If a class had 20 examples to solve on Monday and 28 on Tuesday, what was the per cent of increase?

13. If you have read 78 pages in a book of 300 pages, what per cent of the pages have you read?

14. A man's income is \$3300 a year and he spends \$1914. What per cent of his income does he save?

15. In a certain city 1152 out of the 2400 pupils in school are boys. What per cent are boys?

16. In a certain school 360 out of 750 pupils are girls. What per cent are girls?

17. If a class devotes 42 min. a day to arithmetic one year, and 45 min. a day the next year, what is the per cent of increase per day?

18. If the last chapter of a book is numbered XXXV and you have finished reading Chapter VII, what per cent of the chapters have you read?

19. The purity of gold is measured in carats, or twenty-fourths, 18 carats, or 18 carats fine, meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in a 14-carat ring?

20. What is the per cent of pure gold in a watch case that is 18 carats fine? in a chain that is 10 carats fine? in an ingot of pure gold?

21. If 3 members of a class of 48 have not been either tardy or absent during the year, and 6 members have not been absent, what per cent have not been either tardy or absent? What per cent have not been absent?

22. Stockings are marked 35¢ a pair or 3 pairs for \$1. What per cent on the higher price does a customer save who buys 3 pairs for \$1 instead of paying 35¢ a pair?

Finding the Number of which a Given Number is a Given Per Cent. The third important problem in percentage mentioned on page 17 is to find the number of which a given number is a given per cent.

For example, if a baseball team has lost 9 games, which is 45% of the number of games played, how many games has the team played?

Here we have given the fact that 45% of the number of games is 9. In other words, we have given the product (9) of two factors and one of the factors (0.45), to find the other factor.

Therefore we have $9 \div 0.45 = 20$.

That is, the team has played 20 games.

Check. 45% of $20 = 9$.

Exercise 14. Product and One Factor

Examples 1 to 13, oral

1. If 20% of the inhabitants of a certain city are school children and there are 20,000 school children in the city, what is the total population of the city?

2. If 23% of the inhabitants of a certain city voted at a certain election and there were 2300 ballots cast, what was the total population of the city?

3. A player reached first base 20 times, or $33\frac{1}{3}\%$ of the times he was at bat. How many times was he at bat?

4. An automobile was sold second-hand for \$480, which was 40% of the amount paid for it originally. How much was paid for it originally?

5. If you pay 30% of the expenses of a camping trip and pay \$12, what are the expenses of the trip?

6. In a certain school there are 170 boys, which is 85% of the number of girls. How many girls are there?

Find the numbers of which the following are the given per cents:

7. 72, 8%.	**14.** 222, 3%.	**21.** 175, $87\frac{1}{2}$%.
8. 81, 9%.	**15.** 222, 2%.	**22.** 62.8, $66\frac{2}{3}$%.
9. 72, 12%.	**16.** $87\frac{1}{2}$, $12\frac{1}{2}$%.	**23.** 32.4, $16\frac{2}{3}$%.
10. 75, 15%.	**17.** 99, $3\frac{1}{3}$%.	**24.** 680, 34%.
11. 88, 8%.	**18.** $66\frac{2}{3}$, $33\frac{1}{3}$%.	**25.** 220, 110%.
12. 63, 9%.	**19.** 312, $37\frac{1}{2}$%.	**26.** 2120, 106%.
13. 63, 7%.	**20.** 250, $62\frac{1}{2}$%.	**27.** 34.2, 3.8%.

28. \$6.30 is 15% of what sum of money?

29. \$12.48 is 16% of what sum of money?

30. 23 ft. 10 in. is 22% of what length?

31. If a woman pays \$15.20 for some groceries which is 80% of the list price, what is the list price?

32. A woman set some eggs, and found that 15% of them failed to hatch. If the number failing to hatch was 66, how many eggs did she set?

33. A man saves \$1350.40 a year, which is 32% of his income. How much is his income?

34. A certain school has 20% of its pupils in the sixth grade, which numbers 58. How many pupils are there in the whole school?

35. A man sells some vegetables for \$90, thereby gaining 20% on the cost of raising them. What per cent of the cost of raising the vegetables is the selling price? How much is the cost of raising them?

From the above problems it will be seen that the third important problem of percentage is not nearly so practical as are the first and second problems previously studied.

Exercise 15. Expense Accounts

In the following family expense account for 4 mo., the income being $125 a month, find:

1. The amount saved each month.
2. The total of each item for 4 mo., including savings.
3. The per cent which each total is of the grand total.

Expense Account	Jan.	Feb.	Mar.	Apr.	Total	%
Food	24.10	23.30	24.80	22.15		
Household						
Rent	18.00	18.00	18.00	18.00		
Fuel (average)	4.20	4.20	4.20	4.20		
Gas (cooking)	2.20	2.35	2.15	2.25		
Electricity	1.60	1.80	1.30	1.45		
Help	1.80	2.25	1.30	1.00		
Furnishings	4.20	.65	2.30	9.60		
Personal						
Clothing	7.00	2.00		24.00		
Car fare	.80	1.25	.90	1.20		
Insurance						
Health and accident	1.10	1.10	1.10	1.10		
Life (average)	4.30	4.30	4.30	4.30		
Benevolences						
Church	3.70	4.20	2.60	2.80		
Charity	1.30	2.60	.80	3.20		
Education, Recreation	1.90	2.30	4.70	3.60		
Incidentals	7.30	1.30	2.60	7.38		
Savings						
Total						

These per cents should be carried to the nearest tenth.

Problem Data. The following price list may be used in solving the problems on page 25 and similar problems.

The data may also be secured by the students through inquiry at home or at some grocery. This list may be made the basis of practical problems in simple domestic bookkeeping. The object is, of course, to make arithmetic as real as possible, and when this purpose has been served, the student should proceed to other topics.

Allspice, 10¢ per can; $1 per dozen cans.
Asparagus, 35¢ per can; $4 per dozen cans.
Bacon, American, 28¢ per pound.
Sliced, in jars, 30¢ per pound; $3.25 per dozen jars.
Breakfast cereal, 14¢ per package; $1.60 per dozen packages.
Cinnamon, 10¢ per can; $1 per dozen cans.
Cloves, 30¢ per pound; 50¢ per 2 pound box.
Cocoa, half-pound cans, 25¢; $2.75 per dozen cans.
Coffee, Maracaibo, 20¢ per pound; 5 lb. for 85¢.
Java and Mocha, 35¢ per pound; 5 lb. for $1.60.
Old Government Java, green, 27¢ per pound; 5 lb. for $1.30.
Crackers, Saltines, 25¢ per tin; $2.75 per dozen tins.
Ginger snaps, 8¢ per carton; 90¢ per dozen cartons.
Domino sugar, 5 lb. for 60¢.
Flour, buckwheat, 6¢ per pound; a bag of 24½ lb., $1.30.
Self-raising, 3 lb. for 19¢; 6 lb. for 35¢.
Wheat, 5¢ per pound; $6 per barrel of 196 lb.; 90¢ per sack of 24½ lb.
Granulated sugar, 8¢ per pound.
Herring, 15¢ per can; $1.75 per dozen cans.
Honey, 8-ounce bottles, 30¢; $3.25 per dozen bottles.
Loaf sugar, 11¢ per pound.
Macaroni, 12¢ per package; 25 packages for $2.75.
Maple sirup, pints, 25¢; gallon cans, $1.45; $16.50 per dozen cans.
Olive oil, 40¢ per pint.
Olives, 32¢ per bottle; $3.75 per dozen bottles.
Soups, half-pint cans, 10¢; pint cans, 16¢; quart cans, 28¢; $3.25 per dozen quart cans.
Sugar sirup, half-gallon cans, 50¢; 5-gallon cans, $4.
Tea, Black India, 50¢ per pound.
English Breakfast, 48¢ per pound.

Exercise 16. Household Economics

1. If a family wishes a dozen cans of cocoa, what per cent is saved in buying at the dozen rate?

In such cases reckon the per cent on the higher price.

2. If a family wishes 5 gal. of sugar sirup, how much is saved in buying a 5-gallon can instead of 10 half-gallon cans? What per cent is saved?

3. How much does a hotel manager save in buying 120 gal. of maple sirup by the dozen gallon cans instead of by the single can? What per cent does he save?

4. In which is the per cent of saving greater, in buying honey by the dozen bottles instead of by the bottle, or in buying maple sirup by the dozen cans instead of by the can?

5. If a woman wishes $24\frac{1}{2}$ lb. of buckwheat flour, how much does she save in buying it by the bag?

6. What per cent is saved in buying self-raising flour by the 6-pound package instead of by the 3-pound package?

7. By inquiry at home, make out a grocery list for a week, from page 24. Make two pages of a home account, the left-hand page showing the amount received, and the right-hand page showing the amounts spent for groceries.

8. Make out a bill for six items of groceries, making the proper extensions and footing. Receipt the bill.

Unless the students recall this from their preceding work in arithmetic, the teacher should take it up at the blackboard.

9. If a man uses 2320 cu. ft. of gas in April, how much is his gas bill for that month at 80¢ per 1000 cu. ft.?

10. At the beginning of the month a gas meter registers 14,260, and at the end of the month 17,140. How much is the gas bill for the month, at \$1 per 1000 cu. ft.?

Exercise 17. Heating the House

1. A man put a hot-water heater in his house at a cost of $540, and found that he used 12 T. of coal last season, the coal costing $7.60 per ton. How much did he spend for the heater and fuel?

2. If the house in Ex. 1 was heated for 204 da., what was the average cost of the fuel per day?

3. If the house in Exs. 1 and 2 had 9 rooms, what was the average cost of the fuel per room per day?

4. A man has a steam-heating plant in his house. Last winter it consumed $22\frac{1}{2}$ T. of coal costing $7.25 per ton. How much did the coal cost?

5. If 15% of the coal in Ex. 4 was lost in ashes, how many pounds of coal were lost in ashes?

6. If the house in Ex. 4 has 14 rooms and is heated for 190 da. in a year, what was the average cost of the fuel per day and the average cost per room per day?

7. If 85% of the weight of coal is used in producing heat in a furnace, how many tons of coal are transformed into heat by a furnace that burns 17 T. in a season?

8. A man used 14 T. of coal in his furnace in a season, but on buying a new furnace he used $8\frac{1}{2}$% less coal. At $6.75 a ton, how much did he save on the coal?

9. If a heating plant costing $525 averages 12 T. of coal per year at $7.25 a ton and furnishes the same amount of heat as one costing $375 and averaging 14 T. of coal per year at the same price, which costs the more money in 4 yr., money being worth 5% and the depreciation on each plant per year being 10% of the original cost? How much more?

Exercise 18. The Family Budget

1. Last year Mr. Stone received an income of $3000. He set aside certain per cents of his income as follows: rent, 15%; heat, 3%; light, $1\frac{1}{4}$%; food, 28%; wages, $5\frac{1}{2}$%; incidentals, 7%; other personal expenses, 15%; books, music, church, and pleasure, 8%. How much money did Mr. Stone allow for each of these purposes?

2. Mr. Stone in Ex. 1 really paid for rent, $320; for heat, $52.75; for light, $28.50; for food, $608.75; for wages, $135; for incidentals, $175.50; for other personal expenses, $302.80; and for books, music, church, and pleasure, $167.75. How much did each item of expenditure differ from the estimate and how much did Mr. Stone save during the year?

3. In Ex. 2 what per cent of the amount spent for rent and food was spent for rent and what per cent for food?

4. Mr. Sinclair has an income of $175 a month. He pays during the year for rent, $480; for heat and light, $85.75; for food, $675.80; for clothing, $168.40; for insurance, $54.75; and for other expenses, $250. What per cent of his income does he save?

5. In Ex. 4 what per cent of his income does Mr. Sinclair pay for rent? for food? for heat and light?

6. If a man's income is $225 a month and he spends $600 a year for rent, what per cent of his income does he spend for rent and what per cent is left for other purposes?

7. If a family with an income of $2200 a year spends 16% of its income for rent and 26% for food, what amount does it spend for each of these items?

Students should be encouraged to prepare family budgets at home, with the help of their parents.

Exercise 19. Household Economics

1. A grocer sells coffee in half-pound packages at 19¢ a package and in 4-pound cans at $1.40 a can. If a woman wishes 4 lb., what per cent does she save in purchasing by the can?

2. If you can buy Dutch cocoa in $\frac{1}{4}$-pound boxes at 24¢ a box or in 4-pound cans at $2.65 a can, and you wish 4 lb., what per cent do you save in purchasing by the can?

3. If you can buy maple sirup at 48¢ a quart or in gallon cans at $1.50 a can, and you wish 1 gal. of sirup, what per cent do you save in purchasing by the can?

4. If a woman can buy corn at 15¢ a can or $1.50 per dozen cans, what per cent does she save on 4 doz. cans in buying by the dozen?

5. If a woman can buy soup at 20¢ a can or $2.10 per dozen cans, what per cent does she save on 2 doz. cans in buying by the dozen?

6. A woman can buy a bushel of potatoes for 80¢ or a peck for 25¢. If she needs a bushel of potatoes, what per cent does she save if she buys by the bushel?

7. A woman can buy $\frac{1}{2}$ doz. cans of tomatoes for 75¢ or 1 can for 15¢. If she needs $\frac{1}{2}$ doz. cans, what per cent does she save if she buys by the half dozen?

8. If flour costs $7.40 a barrel (196 lb.) or 5¢ a pound, what per cent does a family save in purchasing flour by the barrel if it requires 196 lb.?

9. If a family's ice bill averages $1.75 a month, and ice costs 35¢ per 100 lb., how many pounds does the family use? If by having a better ice box there is a saving of 10% in the amount of ice used, how many pounds are used? How much is now the average ice bill per month?

10. If you can buy some chairs for \$24 cash or \$3 down and \$3 a month for 8 mo., what per cent do you save if you pay cash?

In Exs. 10–12 interest is not to be considered at this time. It should be mentioned incidentally as a subject to be studied later.

11. If you can buy a sewing machine for \$40 cash or by paying \$4 a month for a year, what per cent do you save if you pay cash?

12. If a reduction of 10% is allowed on all electric-light bills paid before the tenth of each month, what amount would be saved in 4 mo. if advantage is taken of this rule in the account on page 23?

13. After the holidays the price of toys in a certain store was reduced 40%. How much would you save by waiting until after the holidays to buy a mechanical toy that was marked \$3.50 before the reduction?

In the following problems use the current market price as found by inquiry at home or at the store:

14. Find the per cent which you can save in purchasing each of the following in 5-pound packages instead of by the pound: sugar, starch, prunes, raisins.

The teacher may omit Exs. 14–16 if desired. A few such problems, in which the students supply the data, serve, however, to make the subject more real.

15. Find the per cent saved in purchasing each of the following by the dozen cans: tomatoes, corn, peaches.

As a matter of economy it should be noticed that it is not always good policy to purchase in large amounts because the material may deteriorate or be wasted.

16. Find the per cent saved in purchasing potatoes by the bushel instead of by the peck.

Exercise 20. Miscellaneous Problems

1. Mr. Anderson earns $28 a week. He spends 20% of his income for rent, 26% for food for the family, 6% for fuel and lights, 18% for clothing for the family, 10% for church and charity, and 2% for incidentals. How much is left each year for other expenses and for savings?

Although 1 yr. $= 52\frac{1}{7}$ wk., or $52\frac{2}{7}$ wk. in leap years, 52 wk. is always to be taken as a year in problems of this type.

2. The girls in a class in millinery need 20 yd. of a certain quality of ribbon. They can buy this ribbon at 22¢ a yard, or 5 yd. for $1. What per cent will be saved if they take the latter plan?

3. The goods for a certain dress cost $7.80 and the buttons and trimmings cost $2.20. The cost of making the dress is 60% of the cost of the materials. If a dress of like quality and style can be bought for $15, what per cent is saved by buying the dress ready made?

4. Make out a blank like the one shown below, but extended to include your entire school program, and compute the per cent of time devoted to each subject:

Time of Recitation	Subject	Minutes of Recitation	Per Cent of Total	Minutes of Study	Per Cent of Total
9–9 35	Arithmetic	35			
9 40–10 15	English	35			

5. Some girls made 30 pieces of candy from the following recipe: 3 cups granulated sugar, 15¢; $1\frac{1}{2}$ cups milk, 3¢; $\frac{1}{4}$ cake chocolate, $2\frac{1}{2}$¢; an inch cube of butter, 2¢. The fuel cost them 2¢, and they sold the candy at the rate of 3 pieces for 5¢. What per cent was gained on the cost?

Exercise 21. Review Drill

Add, and also subtract, the following:

1.	2.	3.	4.
\$750.68	\$680.01	\$630.27	5 ft. 4 in.
298.98	297.56	429.68	2 ft. 6 in.

Multiply the following:

5.	6.	7.	8.
\$298.63	\$342.80	\$674.39	2 ft. 7 in.
27	92	129	8

Divide as indicated, to two decimal places:

9. \$426.34 ÷ 7. **10.** 3469.1 ÷ 16. **11.** $427\frac{1}{4} \div 0.6$.

12. \$275 is what per cent more than \$200?

13. \$200 is what per cent less than \$275? what per cent less than \$300?

14. If 17 cars cost \$13,600, how much will 9 cars cost?

15. How much is 72% of 350 lb.? of \$3500? of $3\frac{1}{2}$?

16. How much is 175% of \$2500? 12% of \$625? $\frac{1}{2}$% of \$2000? $\frac{3}{4}$% of \$1200? $1\frac{1}{4}$% of \$2400?

Write the answers to the following:

17. 39,987 + 46,296.

18. 73,203 − 59,827.

19. $34\frac{1}{2} \times$ \$42,346.

20. \$29.75 ÷ 25.

21. $\frac{7}{8}$ of 25 ft. 4 in.

22. CDXL = (?).

23. $32\frac{1}{2}$ ft. = (?) yd.

24. $18\frac{1}{2}$ gal. = (?) qt.

25. $4\frac{3}{4}$ bu. = (?) pk.

26. 16 mi. = (?) yd.

In all such drill work the teacher should keep a record of the time required by the students to solve the problems. Each student should strive to improve his record when reviewing the page later in the year.

Exercise 22. Problems without Numbers

1. If you have an account with several items of income and several items of expenses, how do you proceed to balance the account?

2. How would you proceed to make out a household account for a week?

3. How do you find what per cent of the week's income is spent for household expenses?

4. If you know the income of a household and know what per cent of the income is allowed for food, how do you find the amount allowed for food?

5. If you know what fraction of his income a man spends for rent, how do you find what per cent he spends for rent?

6. If you wish to know before you receive the gas bill the amount of gas consumed at your home next month, how will you proceed to read the meter?

7. If you know the cost of tomatoes per dozen cans and the cost per can, how will you find the per cent of saving of a person who purchases a dozen cans at the dozen rate instead of by the can?

8. If you know the recipe for making cake for a certain number of persons, how will you change the recipe if you are making enough for a certain other number of persons?

9. If a man wishes a set of dining-room furniture and finds that, by waiting a week, he can buy it at a mark-down sale at a certain rate per cent off the regular price, how will you find the amount he will save by waiting?

10. If you know how much a man paid for rent last year and how much more he pays this year, how will you find the per cent of increase?

II. ARITHMETIC OF THE STORE

Nature of the Work. Fred Dodge applied for a position in a store. The manager asked him if he could add a column of figures quickly and correctly, and if he could compute quickly in his head. Fred thought he could, but when the manager tested him it was found that Fred was lacking in two things: he had not been taught to check his work, and he did not know the common short cuts in figuring that are used in all stores.

Fred found that the arithmetic work which he needed most was addition, making change, and multiplication. We shall briefly review these subjects.

In this review special attention will be given only to such topics as are not generally treated in the elementary arithmetic which precedes this course.

Oral Addition. In adding two numbers like 48 and 26 mentally, it is better to begin at the left. Simply think of 68 (which is $48 + 20$) and 6, the sum being 74. This is the way the clerk in the store adds $48¢$ and $26¢$.

Exercise 23. Addition

All work oral

Add the following, beginning at the left and stating only the results:

1.	2.	3.	4.	5.	6.
29	68	75	38	45	68
15	23	21	24	25	27
7.	**8.**	**9.**	**10.**	**11.**	**12.**
75	76	88	95	75	80
25	26	75	25	30	75

Exercise 24. Addition

See how long it takes to copy and add these numbers, checking the additions and writing the total time:

1.	2.	3.	4.
$287.75	$349.08	$476.82	$495.53
425.90	346.58	345.46	228.69
381.92	238.46	38.69	642.95
360.58	190.84	248.60	72.38

5.	6.	7.	8.
$458.65	$394.38	$298.05	$482.60
97.86	26.95	342.60	234.65
230.95	700.00	20.07	381.90
48.38	83.56	78.68	32.83
621.04	75.08	380.95	300.00

9.	10.	11.	12.
$2080.60	$3064.45	$4148.20	$3275.25
3679.70	817.66	876.42	842.35
909.36	4239.58	3192.68	3065.06
517.38	86.38	2124.45	2095.05
2310.25	3098.07	629.00	812.40
3096.65	2901.94	5082.00	3028.60

13.	14.	15.	16.
$6240.45	$4063.45	$3083.95	$2438.65
239.76	398.43	498.76	480.00
3865.42	2200.75	298.80	3557.76
396.37	4346.68	3763.84	463.48
900.48	328.93	2989.95	3086.75
1637.00	4043.68	4263.49	2945.50

Oral Subtraction. In subtracting mentally it is better to begin at the left except in making change. In the case of $52-28$ think simply of 32 (which is $52-20$) and take 8 from it, leaving 24.

This subtraction may be treated by the process of making change, next described. Students should be familiar with both processes.

Making Change. If you owe 64¢ to a merchant and give him \$1, he says, "64 and **1** are 65, and **10** are 75, and **25** are \$1," or, briefly, "64, 65, 75, \$1," at the same time laying down 1¢, 10¢, and 25¢.

The merchant will first lay down the coin or coins that will bring the amount up to a multiple of 5; then the largest coin or coins that will bring it up to a multiple of 25; and so on.

Exercise 25. Subtraction

All work oral

Subtract the following numbers:

1.	2.	3.	4.	5.	6.	7.
47	47	47	56	73	83	95
30	33	39	28	34	36	48

Imagine yourself selling goods at a store and receiving in each case the first amount given, the goods costing the second amount. State the amount of change due in each case, and state what coins and bills you would give in change:

8. \$1; 84¢.
9. \$2; \$1.25.
10. \$2; \$1.78.
11. \$3; \$2.20.
12. \$4; \$3.56.
13. \$5; \$2.28.
14. \$5; \$2.65.
15. \$10; \$7.75.
16. \$5; \$2.35.

The teacher should ask the students to find how a cash drawer is arranged, and should describe the cash register. A little work in making change with real or toy money may profitably be given.

Exercise 26. Subtraction

See how long it takes to copy these numbers, to subtract, and to check by adding each difference to its subtrahend; write the total time with the last result:

1.	2.	3.	4.
74,856	24,965	44,430	34,008
36,278	18,986	36,898	30,975
5.	**6.**	**7.**	**8.**
75,500	38,990	78,006	60,900
34,965	29,009	38,869	36,969
9.	**10.**	**11.**	**12.**
$275.68	$220.85	$308.06	$600.04
46.99	165.90	88.79	189.86
13.	**14.**	**15.**	**16.**
$278.00	$470.41	$309.20	$202.70
149.96	82.64	67.64	32.96
17.	**18.**	**19.**	**20.**
$402.64	$300.00	$408.72	$472.92
89.85	183.75	45.86	88.96
21.	**22.**	**23.**	**24.**
$309.92	$482.60	$300.00	$425.30
43.48	193.84	285.68	226.98
25.	**26.**	**27.**	**28.**
$329.80	$408.73	$496.05	$506.00
49.96	229.84	309.78	329.80

Oral Multiplication. When Fred went to work in the store he found that he often needed to multiply rapidly. For example, if he sold 7 yd. of cloth at 45¢ a yard, he needed to find the total selling price at once, without using a pencil. He found that it was usually easier to begin at the left to multiply. In the case of $7 \times 45¢$ he simply thought of $7 \times 40¢$, or \$2.80, and 35¢, making \$3.15 in all.

Exercise 27. Multiplication

Examples 1 to 12, oral

Multiply the following, beginning at the left:

1.	2.	3.	4.	5.	6.
45	38	32	56	56	65
6	4	7	8	9	7
7.	**8.**	**9.**	**10.**	**11.**	**12.**
72	77	56	45	55	35
8	3	4	8	7	8

Multiply the following:

13. 43×473.	**21.** 355×926.	**29.** 35×6464.
14. 38×308.	**22.** 280×628.	**30.** 42×8480.
15. 29×247.	**23.** 84×6088.	**31.** 68×9078.
16. 66×385.	**24.** 29×4756.	**32.** 39×4030.
17. 425×736.	**25.** 63×2798.	**33.** 203×3405.
18. 520×826.	**26.** 42×4802.	**34.** 330×4143.
19. 332×509.	**27.** 34×3006.	**35.** 223×6062.
20. 477×805.	**28.** 23×3989.	**36.** 447×3095.

Teachers who care to give the check of casting out nines may do so at this time. Algebra is required, however, for its explanation.

Short Cuts in Multiplication. You have already learned in arithmetic that there are certain short cuts in multiplication. These short cuts can be used extensively in the store. The most important ones are as follows:

To multiply by 10, move the decimal point one place to the right; annex a zero if necessary.

To multiply by 100 or 1000, move the decimal point to the right two or three places respectively; annex zeros if necessary.

To multiply by 5, multiply by 10 and divide by 2.

To multiply by 25, multiply by 100 and divide by 4.

To multiply by 125, multiply by 1000 and divide by 8.

To multiply by $33\frac{1}{3}$, multiply by 100 and divide by 3.

To multiply by 9, multiply by 10 and subtract the multiplicand.

To multiply by 11, multiply by 10 and add the multiplicand.

Exercise 28. Short Cuts

Find the results mentally whenever you can

Multiply, in turn, by 10, by 100, by 5, by 25, and by 125:

1.	6456	9248	25,192	23,848	22,200
2.	8168	9.376	19,920.8	25.088	58.752
3.	5776	24.432	56,246.4	23.048	46.832

Multiply, in turn, by $33\frac{1}{3}$, by 9, and by 11:

4.	46,977	67,053	15,240	17,604	13,806
5.	441.54	466.74	1639.2	457.05	96.816
6.	483.66	190.56	1804.5	20.166	69.306

Multiply, in turn, by 5, by 25, and by 50:

7.	15,384	56,812	87,824	756.52	73.728
8.	86,988	47,752	93,104	527.24	43.332

Multiply the following:

9. $10 \times \$0.35$.	**20.** $33\frac{1}{3} \times 45$.	**31.** $1000 \times \$7.62$.
10. $10 \times \$225$.	**21.** $33\frac{1}{3} \times 288$.	**32.** $50 \times \$4220$.
11. $10 \times \$7.75$.	**22.** $33\frac{1}{3} \times 585$.	**33.** $125 \times \$3200$.
12. $50 \times \$652$.	**23.** $100 \times \$45$.	**34.** $33\frac{1}{3} \times \$345$.
13. $50 \times \$345$.	**24.** $100 \times \$33$.	**35.** $16\frac{2}{3} \times 186$.
14. $25 \times \$544$.	**25.** $1000 \times \$65$.	**36.** $16\frac{2}{3} \times \$696$.
15. $25 \times \$280$.	**26.** $25 \times \$85.35$.	**37.** $5 \times 40{,}364$.
16. $25 \times \$428$.	**27.** $12\frac{1}{2} \times \$4400$.	**38.** $5 \times \$15{,}680$.
17. $675 \times \$35$.	**28.** $12\frac{1}{2} \times \$4088$.	**39.** $125 \times \$408$.
18. $25 \times \$5.20$.	**29.** $125 \times \$560$.	**40.** $125 \times \$4.08$.
19. $10 \times \$4.80$.	**30.** $125 \times \$5600$.	**41.** $16\frac{2}{3} \times 7200$.

42. How much will 25 books cost at 80¢ each?

43. How much will 25 yd. of cloth cost at 16¢ a yard?

44. How much will 50 cans of corn cost at 14¢ each?

45. How much will 4 books cost at 75¢ each?

46. How much will 25 yd. of cloth cost at 24¢ a yard?

47. How much will $12\frac{1}{2}$ yd. of cloth cost at 48¢ a yard?

48. How much will 80 doz. pencils cost at 56¢ a dozen?

49. How much will 75 books cost at 60¢ each?

50. How much will 75 coats cost at \$5 each?

51. How much will a man earn in 48 wk. at \$25 a week?

52. How will $3\frac{1}{2}$ doz. cans of tomatoes cost at 12¢ a can?

53. At \$7.50 each, how much will 11 tables cost?

54. At \$8.25 each, how much will 9 desks cost?

55. At \$9.60 each, how much will 25 chairs cost?

56. At \$42.50 each, how much will 11 typewriters cost?

Product of an Integer and a Fraction. In the store we frequently have to find the product of an integer and a fraction. For example, we may need to find the cost of $\frac{3}{4}$ yd. of velvet at \$2 a yard. As we have learned,

To find the product of a fraction and an integer, multiply the numerator of the fraction by the integer and write the product over the denominator.

Before actually multiplying, indicate the multiplication and cancel common factors if possible.

Reduce the result to an integer, a mixed number, or a common fraction in lowest terms.

For example, to multiply $\frac{11}{63}$ by 18. Since we have $\frac{11}{63}$, if we have 18 times as much we shall have

$$\frac{\overset{2}{\cancel{18}} \times 11}{\underset{7}{\cancel{63}}}, \text{ or } \frac{22}{7}, \text{ or } 3\tfrac{1}{7}.$$

While we use $\frac{11}{63}$ as an illustration, we seldom find in a store any need for a common fraction with a denominator larger than 8.

Exercise 29. Multiplication

All work oral

Multiply the following, using cancellation when possible:

1. $\frac{2}{3}$ of 6.	**6.** $48 \times \frac{7}{8}$.	**11.** $\frac{7}{12}$ of 48.	**16.** $\frac{7}{8}$ of 960.
2. $\frac{3}{4}$ of 8.	**7.** $50 \times \frac{9}{10}$.	**12.** $24 \times \frac{7}{8}$.	**17.** $\frac{5}{8}$ of 272.
3. $48 \times \frac{5}{8}$.	**8.** $40 \times \frac{3}{10}$.	**13.** $128 \times \frac{3}{4}$.	**18.** $864 \times \frac{7}{8}$.
4. $80 \times \frac{3}{8}$.	**9.** $\frac{4}{5}$ of 35.	**14.** $132 \times \frac{7}{12}$.	**19.** $484 \times \frac{3}{4}$.
5. $50 \times \frac{4}{5}$.	**10.** $\frac{5}{8}$ of 88.	**15.** $\frac{5}{6}$ of 36.	**20.** $192 \times \frac{5}{8}$.

Multiplication of a fraction by a fraction is not so frequently needed in the store as the work given above.

Multiplication involving Mixed Numbers. In the store we frequently need to find such a product as $12\frac{1}{2} \times 16$, as in reckoning the cost of $12\frac{1}{2}$ yd. of cloth at 16¢ a yard. In this particular example we should simply think of $\frac{100}{8}$ of 16, or 200, and state the result at once as \$2. But in general, as we have already learned,

To multiply a mixed number by an integer, multiply separately the integral and fractional parts of the mixed number by the integer and add the products.

For example, $7 \times 2\frac{3}{4} = 14\frac{21}{4} = 19\frac{1}{4}$.

To multiply a fraction by a fraction, multiply the numerators together for the numerator of the product and the denominators together for the denominator of the product.

This case, familiar to the student, is mentioned here for the sake of completeness.

To multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply the results, using cancellation whenever possible.

For example, $2\frac{1}{2} \times 5\frac{3}{4} = \frac{5}{2} \times \frac{23}{4} = \frac{115}{8} = 14\frac{3}{8}$.

Exercise 30. Multiplication

Multiply the following:

1. $15 \times 3\frac{7}{8}$.
2. $18 \times 2\frac{3}{4}$.
3. $36 \times 5\frac{5}{8}$.
4. $80 \times 9\frac{3}{8}$.
5. $48 \times 7\frac{1}{2}$.
6. $36\frac{3}{4} \times 48$.
7. $32\frac{1}{3} \times 45$.
8. $9\frac{3}{8} \times 320$.
9. $9\frac{7}{8} \times 688$.
10. $35\frac{2}{3} \times 18$.
11. $17\frac{3}{5} \times 65$.
12. $26\frac{1}{2} \times 84$.
13. $25\frac{7}{8} \times 320$.
14. $3\frac{7}{8} \times 5\frac{7}{8}$.
15. $3\frac{1}{3} \times 25\frac{7}{8}$.
16. $12\frac{3}{4} \times 15\frac{2}{3}$.
17. $25\frac{5}{8} \times 34\frac{3}{4}$.
18. $14\frac{3}{4} \times 16\frac{2}{3}$.
19. $18\frac{3}{4} \times 23\frac{7}{8}$.
20. $18\frac{1}{3} \times 27\frac{3}{8}$.
21. $15\frac{3}{4} \times 28\frac{3}{8}$.

Use of Aliquot Parts in Multiplication. As you have already learned and would naturally infer from page 38 and from a few examples already met, it is easier to multiply $\$\frac{1}{8}$, $\$\frac{1}{6}$, and $\$\frac{1}{3}$ than it is to multiply $12\frac{1}{2}¢$, $16\frac{2}{3}¢$, and $33\frac{1}{3}¢$. Such parts of a dollar or of any other unit are often called, as we have learned, *aliquot parts.*

At $33\frac{1}{3}¢$ each, 15 books cost 15 times $\$\frac{1}{3}$, or \$5.

At $16\frac{2}{3}¢$ each, 24 rulers cost 24 times $\$\frac{1}{6}$, or \$4.

At $12\frac{1}{2}¢$ each, 16 notebooks cost 16 times $\$\frac{1}{8}$, or \$2.

While goods are seldom marked $16\frac{2}{3}¢$ or $\$\frac{1}{6}$, they are often marked 6 for \$1, which is the same thing.

Exercise 31. Aliquot Parts

All work oral

1. At $12\frac{1}{2}¢$ a yard, how much will 32 yd. of cloth cost?
2. At $16\frac{2}{3}¢$ a yard, how much will 36 yd. of cloth cost?
3. At $33\frac{1}{3}¢$ a yard, how much will 39 yd. of cloth cost?
4. At 8 notebooks for \$1, how much will 24 notebooks cost? How much will 32 notebooks cost?
5. At $16\frac{2}{3}¢$ each, find the cost of 42 glass vases.
6. At the rate of 6 pairs for \$1, how much will 48 pairs of scissors cost?
7. How many children's coats can be cut from $7\frac{1}{2}$ yd. of cloth, allowing $2\frac{1}{2}$ yd. to a coat?

State the products of the following:

8. $9 \times 33\frac{1}{3}¢$.
9. $18 \times 33\frac{1}{3}¢$.
10. $27 \times 33\frac{1}{3}¢$.
11. $150 \times 33\frac{1}{3}¢$.
12. $24 \times 16\frac{2}{3}¢$.
13. $66 \times 16\frac{2}{3}¢$.
14. $48 \times 16\frac{2}{3}¢$.
15. $72 \times 16\frac{2}{3}¢$.
16. $16 \times 12\frac{1}{2}¢$.
17. $48 \times 12\frac{1}{2}¢$.
18. $72 \times 12\frac{1}{2}¢$.
19. $96 \times 12\frac{1}{2}¢$.

Cash Checks. In many of the large stores the clerks are required to fill out cash checks like the following:

Sold by No. 29 Amount rec'd, $40.00				
$2\frac{1}{2}$ yd.	Velvet	1.50	3	75
$14\frac{3}{8}$ "	Satin	.80	11	50
$16\frac{1}{2}$ "	Silk	.95	15	68
Total			30	93
Change due			9	07
			40	00

Exercise 32. Cash Checks

Make out cash checks for the following sales:

1. $3\frac{1}{2}$ yd. cotton @ 18¢, 24 yd. velveteen @ $87\frac{1}{2}$¢, $16\frac{1}{2}$ yd. dimity @ 30¢. Amount received, $30.

2. $8\frac{1}{2}$ yd. gingham @ 30¢, $8\frac{1}{2}$ yd. madras @ 38¢, $9\frac{3}{8}$ yd. silk @ $1.25. Amount received, $20.

3. $24\frac{1}{2}$ yd. linen @ 38¢, $22\frac{1}{8}$ yd. linen suiting @ 85¢, $4\frac{5}{8}$ yd. dimity @ 28¢. Amount received, $30.

4. $6\frac{1}{2}$ yd. India linen @ 42¢, $8\frac{1}{8}$ yd. cheviot @ $1.35, 18 yd. cotton @ $12\frac{1}{2}$¢. Amount received, $20.

5. 14 yd. muslin @ 25¢, $\frac{1}{2}$ yd. velvet @ $3, $6\frac{5}{8}$ yd. India linen @ 45¢, $12\frac{1}{2}$ yd. lining @ 11¢. Amount received, $10.

6. $24\frac{1}{2}$ yd. muslin @ 24¢, $3\frac{1}{8}$ yd. velvet @ $2.40, $26\frac{3}{4}$ yd. lining @ 12¢, $6\frac{3}{8}$ yd. silk @ $1.60, $5\frac{3}{4}$ yd. suiting @ 80¢, $6\frac{1}{2}$ yd. ribbon @ 30¢. Amount received, $50.

The teacher should ask the students to make problems similar to those given above, using the local prices of common materials.

Discount. When goods are sold at less than the marked price, the reduction in price is called *discount.*

Local examples should be mentioned and the students should be asked for illustrations of discount within their experience.

List Price. The price of goods as given in a printed catalogue or list issued by the manufacturer or by the wholesale house is called the *list price.* In stores where the goods are marked, this is called the *marked price.*

Discount is usually reckoned as a certain per cent or as a certain common fraction of the list price or marked price, thus: 20% off, $33\frac{1}{3}\%$ off, $\frac{1}{3}$ off, and so on.

Net Price. The price of goods after the discount has been taken off is called the *net price.*

Cash Discount. A discount allowed because the purchaser pays at once is called a *cash discount.*

For example, a Boy Scout suit may be marked \$6, but owing to the desire of the dealer to clear out his stock at the end of the season he may mark it to sell for 10% off for cash. The suit will then be marked \$6 less 10% of \$6, or \$6 − \$0.60, or \$5.40.

Trade Discount. When merchants, jobbers, or manufacturers sell to dealers they often deduct a certain amount from the list price. This reduction is called a *trade discount.*

Such terms as *retail dealer*, *wholesale dealer* or *jobber*, and *manufacturer* should be explained by the teacher if necessary.

A special form of trade discount is allowed for very large orders. This is called a *quantity discount.*

The terms of discount are often stated thus: 2/10, 1/30, *N*/60, these symbols meaning 2% discount if the bill is paid within 10 da., 1% if paid within 30 da., net (no discount) thereafter, and the bill to be paid within 60 da.

Exercise 33. Discounts

Examples 1 to 15, oral

1. If some goods are marked \$20, and 10% discount is allowed, what is the selling price?

2. If a book marked 80¢ is sold at a discount of 25%, this is how many cents below the marked price?

3. If a merchant buys \$800 worth of goods and is allowed 10% discount in case he pays for them at once, how much does he save by prompt payment?

Find the discounts on the following at the rates specified:

4. \$80, 10%.
5. \$25, 10%.
6. \$50, 20%.
7. \$25, 20%.
8. \$40, 25%.
9. \$88, 25%.
10. \$60, 50%.
11. \$90, 50%.
12. \$120, 10%.
13. \$150, 20%.
14. \$160, 25%.
15. \$250, 50%.

16. If goods marked \$475 are sold to a dealer at a discount of 20%, how much do they cost him?

17. If a merchant marks a lot of suits at \$24.75 each, and sells them at $\frac{1}{3}$ off, what is the net price of each?

Students should be asked to watch advertisements in the newspapers to see what discounts are offered and should state the probable reasons for these discounts.

Given the marked prices and rates of discount as follows, find the net prices:

18. \$17.50, 10%.
19. \$16.50, 20%.
20. \$27.75, 20%.
21. \$64.40, 25%.
22. \$86.60, 25%.
23. \$21.60, $12\frac{1}{2}$%.
24. \$72.30, $33\frac{1}{3}$%.
25. \$38.70, $33\frac{1}{3}$%.
26. \$43.50, $16\frac{2}{3}$%.
27. \$86.40, $16\frac{2}{3}$%.
28. \$48.60, 25%.
29. \$27.70, 50%.
30. \$35.00, 10%.
31. \$34.75, 20%.
32. \$65.25, 20%.

Several Discounts. In some kinds of business two or more discounts are frequently allowed. For example, a dealer may buy hardware listed at \$200 with discounts of 20%, 10% (20% and 10%, often called and written simply 20, 10). This means that 20% is first deducted from the list price, and then 10% from the remainder.

The list price is \$200.
The list price less 20% is \$160.
Then \$160 less 10% is \$144, the *net price*.

In reckoning discounts the student should discard every fraction of a cent in the several discounts.

Exercise 34. Several Discounts

Examples 1 to 5, oral

1. From \$100 take 10%, and 5% from the remainder.
2. From \$800 take 25%, and 1% from the remainder.
3. From \$600 take 20%, and 20% from the remainder.
4. A dealer bought some goods at a list price of \$100, with discounts of 10%, 10%. How much did he pay?

Find the net prices of goods marked and discounted as follows:

5. \$400, 20%, 30%.
6. \$1400, 35%, 5%.
7. \$800, 20%, 15%.
8. \$1200, 12%, 4%.
9. \$1300, 15%, 10%.
10. \$800, 20%, 10%.
11. \$600, 15%, 10%.
12. \$550, 10%, 10%.

13. What is the difference between a discount of 50% on \$1000, and the two discounts of 25%, 25%?

14. Is there any difference between a discount of 5%, 4%, and one of 4%, 5%, on \$900? How is it on \$600?

Sample Price List. The following is a price list of certain school supplies, with the discounts allowed to schools and dealers when the prices are not net:

Composition books,	\$4.80 per gross, less 5%
Drawing compasses,	1.75 per doz., less 10%, 5%
Drawing paper, 9×12,	1.30 per package, less 10%
Drawing pencils,	4.80 per gross, less 20%
Penholders,	3.40 per gross, less 12%, 4%
Pens,	0.65 per gross, less 25%, 10%
Rulers,	0.40 per doz., net
Thumb tacks,	0.45 per 100, less 30%
Tubes of paste,	4.15 per gross, less 10%, 6%

Exercise 35. Purchases for the School

1. A school board wishes to buy 8 packages of drawing paper and 200 thumb tacks. How much will they cost?

In this exercise use the above price list.

2. How much will 12 gross of pens and 2 gross of composition books cost? 4 gross of pens and 3 doz. rulers?

3. There are 18 students in a class, and each student needs compasses and a ruler. How much will all these drawing instruments cost the school?

4. If a dealer sells pens at a cent apiece, how much does he gain per gross? If he sells penholders at 3¢ each, how much does he gain per gross?

5. A dealer buys 3 gross of rulers and 2 gross of drawing pencils. He sells the rulers and pencils at 5¢ each. How much does he gain in all?

6. If a dealer buys 2 gross of tubes of paste for mounting pictures and sells the tubes at 5¢ each, how much does he gain on the purchase? how much per gross?

Bill with Several Discounts. The following is a common form of a jobber's bill of goods with several discounts:

Burlington, Ia., May 13, 1920

Mr. Geo. S. Miller, Cedar Rapids, Ia.

Bought of ROBERTS & STONE, Jewelers
1072 Passaic Avenue

Terms: *20%, 10%*

May	*10*	*8 doz. spoons @ $14.75*	*118*	*00*		
		7 doz. plated forks @ $4.20	*29*	*40*	*147*	*40*
		Less 20%, 10%			*41*	*27*
					106	*13*

Exercise 36. Bills

Make out bills for the following:

1. 4 doz. sweaters at $34. Discounts 10%, 5%.

2. 16 doz. files at $7.30. Discounts 25%, 20%.

3. 625 yd. taffeta at $1.35; 240 yd. velvet at $1.80. Discounts 15%, 10%.

4. 6 doz. pairs hinges at $4.50; 12 doz. table knives at $9.20. Discounts 20%, 10%.

5. 16 doz. locks at $4.50; 4 doz. mortise locks at $4.85. Discounts 20%, 8%.

6. 840 yd. taffeta at $1.10; 12 gross pompons at $150; 4 doz. pieces braid at $21.60. Discounts 10%, 5%.

7. 960 yd. silk at $1.75; 640 yd. lawn at 27¢; 860 yd. taffeta at $1.05. Discounts 10%, 5%, 5%.

Receipted Bill. The following is a receipted bill for some goods purchased by a retail merchant from a jobber:

Chicago, Ill., Dec. 17, 1920

Mr. A. M. Nourse, Joliet, Ill.

Bought of STARR & TIFFANY, Jewelers
8378 Burlington Ave.

Terms: *2%*

Dec.	*2*	*3 doz. silver forks @ $22*	*66*				
		1/6 doz. salad forks @ $24	*4*		*70*		
		Less 2%			*1*	*40*	
					68	*60*	
		Received payment					
		Starr & Tiffany					

In this case Mr. Nourse is the *debtor* (Dr.), since he is in debt for the amount; the firm of Starr & Tiffany is the *creditor* (Cr.), since it trusts Mr. Nourse, or gives him credit. On this bill only a single discount was allowed.

A receipt may also be written separately instead of appearing on a bill. Such a receipt should be dated, and should be in substantially this form: "Received from —— the sum of —— dollars for ——." The receipt should be signed by the creditor.

The subject of commercial discount is of great value because of its extensive use not only in wholesale transactions but even in bargain sales. Students should understand that some of the reasons for allowing discounts are buying in large quantities and paying cash down or within a specified time, and they should become familiar with the ordinary deductions from list prices. It is well, for obvious reasons, to consider bills and receipts, preferably of a local character, in connection with the study of this topic.

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Invoice. A bill stating in detail a list of items and prices of goods sold is called an *invoice*. A sample invoice follows:

St. Paul, Minn., Dec. 12, 1920

Mr. James P. Dunbar, Fargo, N.D.

Bought of RANDALL BROTHERS

IMPORTERS OF DRY GOODS AND FANCY GOODS

Terms: *10 da.*

236	14	pc. blue flannel 40, 41, 41^1, 41^3, 40^2, 40, 41, 42, 41^3, 42^1, 41, 42^2, 41, 40	576	33⅓¢	192		
427	8	pc. silk 40, 39, 41^2, 40^3, 39^1, 41, 42^2, 40	324	$1.25	405		597

In the above invoice 41^1 means $41\frac{1}{4}$; 41^2 means $41\frac{2}{4}$, or $41\frac{1}{2}$; and 41^3 means $41\frac{3}{4}$. This is the customary way of indicating the number of yards in pieces of goods. The numbers 236 and 427 refer to the price list in which the goods are described. The numbers 14 and 8 indicate the number of pieces (pc.) bought. The number of yards of the first is 576 and of the second 324.

The expression "Terms: 10 da." means that Mr. Dunbar has 10 da. in which to pay for the goods. Such an invoice may or may not mention the discount allowed.

There is no essential arithmetic difference between a bill of goods and an invoice of a wholesale dealer.

Exercise 37. Invoices

Make out invoices for the following:

1. 24 doz. caps @ \$15.50, $2\frac{1}{2}$ doz. ties @ \$8.40.

2. 32 pc. ribbon @ \$1.25, 14 pc. @ \$1.30, 48 pc. @ \$1.75, 24 pc. @ \$1.12$\frac{1}{2}$, 32 pc. @ \$1.37$\frac{1}{2}$, 48 pc. @ \$1.25, 36 pc. @ \$1.40, 64 pc. @ \$1.60.

3. 3 carloads coal, 21,700 lb., 24,200 lb., 25,100 lb., @ \$5.20 per short ton; 3 carloads coal, 22,700 lb., 21,900 lb., 20,400 lb., @ \$5.75 per short ton. Terms: 4% for cash.

4. 1 gro. cans sardines @ \$3.40 per doz., 9 doz. cans shrimps @ \$1.75, 8 doz. tins herrings @ \$2.50, $6\frac{1}{4}$ doz. cans lobster @ \$2.88, 32 cans mackerel @ $16\frac{2}{3}\phi$, $4\frac{1}{2}$ doz. cans kippered herrings @ \$2.65. Terms: 4% for cash.

5. 8 doz. packages codfish @ \$1.80, 9 doz. cans salmon @ \$2.40, 15 doz. cans caviar @ \$3.50, 18 doz. cans Yarmouth bloaters @ \$2.40, 24 doz. cans tongue @ \$8.75, 16 doz. cans baby mackerel @ \$1.80. Terms: $3\frac{1}{2}$% for cash.

6. 12 doz. jars meat extract @ \$3.40, 24 doz. cans chicken @ \$3.15, 24 doz. cans beef @ \$2.40, 18 doz. cans soup @ \$3.40, 9 doz. cans clam chowder @ \$3.75, 16 doz. cans clam juice @ \$1.10. Terms: 3%, 2%.

7. 8 armchairs @ \$6.75, 24 kitchen chairs @ \$1.25, 12 kitchen tables @ \$2.25, 6 bedroom sets @ \$42.50, 24 rockers @ \$8.25, 16 dining tables @ \$12.50, 9 sideboards @ \$16.66$\frac{2}{3}$. Terms: 6%, 4%.

8. 20 pc. linen containing 40^1, 40^2, 38^3, 42, 41^1, 40^2, 40, 40, 40^1, 41^2, 43, 39^3, 40^1, 42^1, 40^2, 40^1, 41^3, 42, 39^1, 40 yd., @ 75ϕ; 16 pc. silk containing 41^1, 42^1, 40, 39^2, 42, 40^3, 38, 41^1, 43, 41^3, 39, 40^2, 40^1, 41^1, 40, 39^2 yd., @ \$1.40. Terms: 6%, 3%.

Exercise 38. Review

1. Make out an invoice for the following goods purchased May 7 and paid for May 10, terms 2/10, 1/20, *N*/30: 10 bolts dress linen, 10, 12, 11^1, 12^2, 10^3, 11^3, 12, 10^1, 11^2, 11^3 yd., @ 48¢ a yard; 24 bolts French nainsook, 12 yd. each, @ 18¢ a yard; 30 bolts mercerized lingerie batiste, 24 yd. each, @ 22¢ a yard; 32 bolts imported lawn, 10 yd. each, @ 46¢ a yard. Insert names and find the net cost.

2. In Ex. 1 find the net cost if the payment is made on May 20.

3. Make out an invoice for the following goods purchased Sept. 20 and paid for Oct. 5, terms 2/20, *N*/90: 5 No. 264 plows @ $42.50 less 20%; 3 No. 178 self-dump hayrakes @ $18.60 less 15%; 9 No. 325 hay stackers @ $46.50 less 15%. Insert names and find the net cost.

4. A jobber offers his customers discounts of 15%, 10%, but the invoice clerk made a mistake on a bill of $85 and gave a single discount of 25%. How much did the error cost the clerk or the jobber?

5. A manufacturer lists a desk at $52 less 25%, and a rival manufacturer offers a similar desk for $57 less $\frac{1}{3}$. If the first dealer increases his discount to 25%, 3%, which will be the lower net price and how much lower?

6. A retail dealer being allowed a discount of 20%, 2%, find the net price of the following goods purchased from a wholesale dealer at the list prices stated: 1 dining table, $34; 8 chairs @ $2.85; 1 buffet, $32.50; 1 rug, $26.

7. A dressmaker bought the following bill of goods, receiving a trade discount of 8% and a cash discount of 5%: $8\frac{1}{2}$ yd. broadcloth @ $3; $13\frac{1}{2}$ yd. silk lining @ 80¢; 3 yd. trimming @ $2.40. What was the net price?

Exercise 39. Review Drill

1. Multiply 488 by $\frac{1}{2}$; by 0.5; by 50%; by 50.
2. Multiply 24 by $33\frac{1}{3}\%$; by 25%; by $12\frac{1}{2}\%$.
3. Multiply $\frac{2}{3}$ by $\frac{3}{4}$; $\frac{2}{3}$ by $\frac{3}{8}$; $\frac{4}{5}$ by $\frac{3}{4}$.
4. Divide $\frac{1}{2}$ by $\frac{1}{4}$; $\frac{1}{4}$ by $\frac{1}{2}$; $\frac{3}{4}$ by $\frac{1}{2}$; $\frac{1}{2}$ by $\frac{3}{4}$.
5. Multiply 8 by $\frac{1}{8}$; 8 by 0.125; 8 by 125.
6. Divide 4800 by 100; by 10; by 20; by 40.
7. Add $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{2}$ and $\frac{3}{4}$; $\frac{5}{8}$ and $\frac{3}{4}$.
8. Find the values of $1-\frac{5}{8}$; $1\frac{1}{8}-\frac{5}{8}$; $5\frac{1}{4}-3\frac{3}{4}$.
9. Express as decimals and also as common fractions: $37\frac{1}{2}\%$; $16\frac{2}{3}\%$; $62\frac{1}{2}\%$; $66\frac{2}{3}\%$; $87\frac{1}{2}\%$; $83\frac{1}{3}\%$; $33\frac{1}{3}\%$.
10. Express 12 ft. as inches; 96 in. as feet; 69 ft. as yards; 51 yd. as feet; 16 gal. as quarts; 16 qt. as gallons; 3 lb. 4 oz. as ounces; 32 oz. as pounds.
11. \$500 is what per cent of \$400?
12. \$500 is what per cent more than \$400? than \$250?
13. \$400 is what per cent less than \$500? than \$800?
14. If 8 typewriters cost \$500, how much will 15 cost?
15. If 9 desks cost \$63, how much will 36 desks cost?
16. If the rent of an apartment is \$720 for $\frac{3}{4}$ yr., how much is the rent for a year?

Perform the following operations:

17. $41\frac{2}{3}$ in. $+1\frac{1}{2}$ in.
18. $5\frac{3}{8}$ in. $-2\frac{3}{4}$ in.
19. $2\frac{3}{4}\times 27\frac{1}{2}$ in.
20. $6\frac{7}{8}$ ft. $\div 3\frac{7}{16}$ ft.
21. $3.75\div 2\frac{1}{2}$.
22. 25% of \$1.60.
23. $\$2.75\div 1\frac{1}{4}$.
24. $\frac{2}{3}$ of $34\frac{1}{2}$ ft.
25. $\frac{7}{8}$ of 9 ft. 4 in.
26. 75% of 6 lb. 4 oz.

Exercise 40. Problems without Numbers

1. If you buy a fishing rod and sell it at a certain rate per cent above cost, how do you find the selling price?

2. If a man buys a car and sells it at a certain per cent below cost, and you know how much he paid to keep the car and how much he saved by using it, how do you find whether he gained or lost, and the amount?

3. If you know the number of school days in the year and the number of times you were absent from school during the year, how do you find the percentage of absences?

4. If you know the number of times and the percentage of times a train arrives on time during a certain month, how do you find the number of runs the train makes?

5. If you sell a person a certain bill of goods, and he hands you more than the required amount of money, how do you proceed to make change?

6. If you know a man's income and the amount which he pays for rent, how do you find what per cent of his income he pays for rent?

7. If you know the rate and amount of an agent's commission, how do you find the selling price?

8. How do you express a given per cent as a decimal? as a common fraction?

9. If you know what a certain per cent of a number is, how do you find the number?

10. How do you find what per cent one given number is of another given number?

11. If you sell a man a number of articles at a certain price each, and this price is an aliquot part of a dollar, what is the shortest method of finding the amount due?

III. ARITHMETIC OF THE FARM

Nature of the Work. The farming industry is one of the largest in our country. There are between six and seven million farms in the United States and their total area is nearly 900,000,000 acres. These farms are worth, with their buildings and machinery, over $40,000,000,000 and they produce about $10,000,000,000 annually. From these immense sums it will be seen how important is the farming industry and how necessary it is to know some of the problems relating to it.

Every boy and every girl who lives on a farm ought to know how to measure a field and find its area, how to keep farm accounts, and how to make the necessary computations relating to the dairy, the crops, and the soil. Even boys and girls who live in villages and cities should, for their general information, know something of these matters, just as those who live on the farm should know something about the problems of the city.

The work of measuring land and computing such volumes as the farmer uses is taken up in the geometry in this book, but the familiar case of the area of a rectangle is assumed to be understood.

Teachers in village and city schools will find in the following pages a sufficient number of problems for their purposes, but the entire topic may be omitted if necessary. In rural schools, however, additional problems should be drawn from the locality in which each school is placed. Agricultural products, the soils, the customs, the wages, and the prices all vary greatly in different sections of our country, and the teacher should encourage the students to bring to school problems which relate to local interests. Problems about irrigation are important in some states, while in others they are quite unheard of; the alfalfa crop is very important in certain parts of the country, while in others it is not; land is laid out in sections in some states, while it is never so laid out in others. The teacher should be guided by a knowledge of these various customs.

Exercise 41. Cost of Wastefulness

1. If a farm wagon that cost \$60 is left out in the yard instead of being kept in the shed, it will last about 6 yr., but if kept under cover, it will last about twice as long. What per cent of the cost, not considering the interest on the money, does a farmer pay for his carelessness per year if he leaves the wagon out of doors?

2. A farmer after threshing his wheat had 16 T. of straw left. A ton of this straw contains 10 lb. of nitrogen worth 15¢ a pound, 18 lb. of potassium worth 6¢ a pound, and 2 lb. of phosphorus worth 12¢ a pound. If the farmer wastes the straw instead of using it on the soil as fertilizer, how much money does he waste?

3. It is computed that a certain kind of farm machinery depreciates in value as follows, if reasonably good care is taken of it: 10% of the original value the first year, 8% of the original value the second year, 6% of the original value the third year, 3% of the original value the fourth year and each year thereafter. A machine of this kind cost a farmer \$240. He did not take proper care of it, and at the end of 4 yr. it was worth only \$115. The per cent of the original value thus lost in the 4 yr. was how much more than the per cent that would have been lost had proper care been taken of the machine?

4. The strip of waste land along each side of a woven-wire fence is 2 ft. 6 in. wide, the strip along a barbed-wire fence is 3 ft. wide, and the strip along a rail fence is 4 ft. 6 in. wide. How many rods of each kind of fence cause a waste of 1 acre of land on one side? At \$90 an acre, compare the losses occasioned by the various types of fences for the inside of a rectangular field 40 rd. wide and 60 rd. long fenced in on the four sides.

Farm Accounts. Many careful farmers keep systematic accounts of the receipts and expenditures for each of their fields, as well as for the farm as a whole. In the problems of the following exercise an itemized statement is given of expenditures for a 20-acre field of corn.

Exercise 42. Farm Accounts

1. In the following account supply the missing amounts:

Expenses					
Apr.	9	Plowing, $5\frac{1}{2}$ da.	@ $4.80		
	11	Harrowing, $2\frac{1}{2}$ da.	@ $4.50		
	29	2 bu. seed corn	@ $1.00		
	30	Planting, 2 da.	@ $4.80		
May	16	Replanting, 1 da.	@ $1.25		
	17	Harrowing, 2 da.	@ $4.50		
	23	Plowing, 4 da.	@ $4.75		
June	9	Plowing, 3 da.	@ $4.75		
	28	Plowing, 3 da.	@ $4.75		
July	15	Plowing, 3 da.	@ $4.75		
Sept.	19	Cutting, 6 da.	@ $4.00		
Nov.	8	Husking, 718 bu.	@ 4¢		
	20	Rent of land @ $5.50 per acre			
		Total			

2. If the receipts in the above case came from the sale of 718 bu. of corn at 62¢, with $40 worth kept on hand, the expenditures are what per cent of these receipts?

3. In the field of Exs. 1 and 2 find the net profit.

In rural schools it is desirable to secure or to have the students secure local accounts of this kind. This is the best way to make the subject real to those who are studying it.

Exercise 43. Farm Records

1. Ralph's father explained to him what was meant by grading the cows, keeping their records for milk, and selling the poor cows. He showed him this farm record:

With Systematic Grading			Without Systematic Grading		
Herd	Annual cost of food per cow	Annual profit per cow	Herd	Annual cost of food per cow	Annual profit per cow
a	$34.28	$34.02	*g*	$31.65	$13.34
b	33.37	36.19	*h*	40.58	8.13
c	47.11	25.83	*i*	38.30	22.22
d	36.72	38.27	*j*	37.40	22.33
e	31.19	48.86	*k*	33.78	38.93
f	31.57	42.81	*l*	32.88	14.91

Ralph and his father then figured out the average annual cost of food and profit per cow, in each class of herds. They did this by dividing by 6 the total of each of the four columns. What were the results?

2. At a certain experiment station the five most profitable and the five least profitable cows compared as follows:

Grade of Cows	Cost of Feed and Care Annually	Pounds of Butter Fat Annually	Average Cost of 1 lb. of Butter Fat
Five most profitable cows	$56.54	304	
Five least profitable cows	52.02	189	

Compute the average cost of 1 lb. of butter fat for the most profitable and for the least profitable cows.

Exercise 44. Problems of the Dairy

1. A farmer sells 26,250 lb. of milk to a creamery in a certain month. The milk averages 4.2% by weight of butter fat. With how many pounds of butter fat does the creamery credit the farmer in that month?

2. If 6 lb. of butter fat are needed in making 7 lb. of butter, what is the value of the butter produced from 1236 lb. of butter fat, the butter being worth 34¢ a pound?

3. A certain dairy sells to a creamery milk averaging 3.75% of butter fat. The butter fat weighs 630 lb. How many pounds of milk does the dairy sell?

4. A farmer has two cows, one supplying 986 lb. of milk testing 3.1% butter fat in a certain month, and the other 812 lb. testing 4.2%. If the creamery allows the farmer 32¢ a pound for butter fat, which cow pays him the more and how much more, the feed and care costing the same?

5. A creamery uses 7500 lb. of milk in a week. The skim milk amounts to 80% of the whole milk and contains 0.6% butter fat. How many pounds of butter fat are lost in the skim milk?

6. A farmer owns a herd of 18 cows that average 25 lb. of milk per head daily. This milk tests 3.5% butter fat, and the butter fat is worth 26.5¢ per pound. How much does the farmer receive in 30 da. for the butter fat?

7. A herd of 24 cows averages 22 lb. of milk per cow daily, and another herd of 18 cows averages 28 lb. per cow. The milk of the first herd averages 5% butter fat and that of the second herd 3.5%. How many more pounds of butter fat are produced by the first herd per week?

In rural communities there should be special computations on such subjects as rations, cost of labor, and the income from cows.

8. Given the following table showing the number of pounds of nitrogen, phosphorus, and potassium in 1000 lb. of each of five kinds of feed, find the per cent of each of these three ingredients in each of the five kinds:

Feed	Nitrogen	Phosphorus	Potassium
Wheat straw . .	5.0	0.8	9.0
Timothy hay . .	12.0	1.5	11.8
Clover hay . . .	20.0	2.5	15.0
Corn	17.1	3.0	3.4
Wheat	23.7	4.0	4.3

9. The following table shows the amount of protein and carbohydrates in certain kinds of feed:

Feed	Weight of a Bushel in Pounds	Pounds per Bushel	
		Protein	Carbohydrates
Rye	56	5.0	39
Barley	48	4.0	32
Corn	56	3.5	40
Oats	32	3.0	19

A dairy cow of average size requires daily about 2 lb. of protein and 12 lb. of carbohydrates. When corn is 62¢ and oats 41¢ per bushel, which is the cheaper food, considering the protein alone? considering carbohydrates alone?

10. In Ex. 9 the weight of the protein in a bushel of rye is what per cent of the weight of the rye? Answer the same question for barley: for corn: for oats. Answer the same questions for the carbohydrates.

The weight of a bushel varies in the different states.

Exercise 45. Feeding Corn

1. When corn was selling at 55¢ a bushel, a farmer decided to feed his corn to his cattle. He estimated that the increase in the value of the cattle, from the corn alone, was 60¢ for each bushel used for feed. What was the per cent of increase in the value of the corn by using it as feed?

2. A record of the result of feeding corn to hogs was kept on several farms. On one farm, when corn was selling at 55¢ a bushel, it was found that the increase in the value of the hogs was equivalent to 82¢ per bushel of corn fed to hogs. What was the per cent of increase in the value of the corn by using it as feed?

3. On another farm the figures of Ex. 2 were 52¢ a bushel for corn when sold and 75¢ a bushel when used as feed. What was the per cent of increase in the value of the corn by using it as feed?

4. A bushel of corn contains about $\frac{2}{3}$ lb. of nitrogen, $\frac{1}{4}$ lb. of phosphorus, and $\frac{1}{4}$ lb. of potassium. How many pounds of each of these substances are contained in the crop from a 20-acre field yielding 62 bu. of corn to the acre?

5. If it costs $11.80 per acre to grow a crop of corn and haul it to the market, where it is sold at 60¢ per bushel, what is the net profit from a 25-acre field if the rent on the land is $9 per acre and the land yields an average of 58 bu. of corn per acre?

6. A cow is fed daily 6.5 lb. of corn worth 60¢ per bushel of 56 lb. What will be the cost of the corn fed to the cow in 2 mo. of 30 da. each?

7. If a bushel of corn, when fed to hogs, will produce 9.5 lb. of pork, how much will it cost to produce 1 lb. of pork when corn is 62¢ a bushel?

Exercise 46. Farm Income

1. A farmer owns two farms of the same size and value. One he runs himself and the other he lets on shares. The following table shows the itemized income of each farm:

	Home Farm	Rented Farm
Dairy products	$348.60	$103.75
Wool	43.75	28.80
Eggs and poultry	316.80	123.50
Domestic animals	637.50	321.60
Crops	1072.80	785.30

Find the total receipts on each farm.

2. In Ex. 1 find the per cent of increase of each item of income on the home farm over the corresponding item on the rented farm.

3. The following table shows the itemized expenses of the two farms mentioned in Ex. 1:

	Home Farm	Rented Farm
Labor	$212.60	$140.30
Fertilizers	92.30	12.00
Feed	63.50	58.60
Maintaining buildings	31.75	63.40
Maintaining equipment	15.50	42.50
Taxes and miscellaneous	82.50	80.75

Find the total expenses of each farm, the difference in each pair of items, the difference in the total expenses, and the net gain of each farm.

4. In Ex. 3 find what per cent more was paid for labor on the home farm than on the rented farm.

5. In Ex. 3 find what per cent more was paid for maintaining buildings on the rented farm than on the home farm.

Exercise 47. Soils and Crops

1. The soil of an acre of rich land in the Corn Belt, plowed to the depth of $6\frac{2}{3}$ in., is estimated to weigh 2,000,000 lb. and to contain 8000 lb. of nitrogen, 2000 lb. of phosphorus, 35,000 lb. of potassium, and 15 T. of calcium carbonate (limestone). Express each of these weights as per cent of the total weight.

The figures given in the problems on this page are, as usual in such cases, only approximations, because soil and crops vary greatly in different places. The figures are, however, always based upon scientific results as obtained in agricultural experiment stations.

2. The plant food liberated from the soil during an average season is 2% of the nitrogen, 1% of the phosphorus, and $\frac{1}{4}$% of the potassium contained in the surface stratum of $6\frac{2}{3}$ in. mentioned in Ex. 1. Find the number of pounds of each of these elements liberated at these rates from a 100-acre field in 1 yr.

3. The grain in a 100-bushel crop of corn takes from the soil 100 lb. of nitrogen, 17 lb. of phosphorus, and 19 lb. of potassium, and the stalks take 48 lb., 6 lb., and 52 lb. respectively. Express each of the first three as per cent of the total weight of the corn, allowing 56 lb. to the bushel. Allowing 60 lb. of stalks to produce 1 bu. of shelled corn, express each of the last three as per cent of the total weight of the stalks.

4. Given that 1 T. of clover hay contains 40 lb. of nitrogen, 5 lb. of phosphorus, and 30 lb. of potassium, express each as per cent of the total weight of the hay.

5. If 50 bu. of wheat, weighing 60 lb. per bushel, contains 12 lb. of phosphorus, 13 lb. of potassium, 4 lb. of magnesium, 1 lb. of calcium, and 0.1 lb. of sulphur, each is what per cent of the weight of the wheat?

Exercise 48. Good Roads

1. A teamster had to haul $7\frac{1}{2}$ T. of barbed wire a distance of 13 mi. over poor roads from the railway. He found that he could haul only 1500 lb. to a load and that it took him a full day to make the round trip. How long did it take him to haul the $7\frac{1}{2}$ tons of wire, and how much did it cost at \$5 per day for man and team?

2. In Ex. 1 what was the cost of hauling 1 ton 1 mi., or the cost of 1 ton-mile, as it is ordinarily called?

3. In Ex. 1, after a new state road had been constructed, the teamster found that with the same team he could haul $2\frac{1}{2}$ T. to the load and make the round trip in 1 da. How much did it then cost to haul a ton of wire? What was the cost per ton-mile?

4. Comparing the results in Exs. 2 and 3, what per cent less was the cost per ton-mile in Ex. 3, owing to good roads? What was the per cent of time saved in hauling $7\frac{1}{2}$ T.?

5. A farmer lives 10 mi. from the railway. The road was formerly so bad that with a two-horse team he could haul only 30 bu. of wheat to the load, and it took 1 da. to make the round trip. At \$5 a day for man and team, how much did it cost per bushel to haul the wheat?

6. The roads were recently improved. The farmer can now haul 75 bu. to the load. Allowing $\frac{3}{4}$ da. for the trip, find the cost per bushel of hauling the wheat now.

7. Comparing Exs. 5 and 6, how much more money due to improved roads does the farmer get per bushel?

8. Taking 60 lb. as the weight of a bushel of wheat and comparing Exs. 5 and 6, what has been the per cent of reduction in cost of cartage per ton-mile?

Exercise 49. Review Drill

1. Express 0.4% as a decimal fraction.

2. Express 2.8 as per cent.

3. Express $2\frac{1}{2}$ qt. as per cent of 1 gal.; of 5 gal.

4. Express an inch as per cent of 1 yd.; of 3 yd.

5. Express $8\frac{1}{3}\%$, $16\frac{2}{3}\%$, $33\frac{1}{3}\%$, $62\frac{1}{2}\%$, $66\frac{2}{3}\%$, $83\frac{1}{3}\%$, and $87\frac{1}{2}\%$ as common fractions in lowest terms.

Find the value of each of the following:

6. 50% of 860.

7. 25% of $62\frac{1}{2}$.

8. $37\frac{1}{2}\%$ of \$9600.

9. $133\frac{1}{3}\%$ of \$19.56.

10. $187\frac{1}{2}\%$ of \$19.28.

11. 0.7% of 275 lb.

12. 0.08% of 56,000.

13. 225% of 4800.

14. 2.25% of 4800.

15. 300% of 156.7.

Multiply as indicated:

16. 275×3468. **17.** 39.6×31.78. **18.** 0.432×687.2.

Divide to two decimal places:

19. $1 \div 3.7$. **20.** $0.27 \div 0.5$. **21.** $68.01 \div 0.7$.

22. \$30 is what per cent of \$360? of \$3600? of \$36,000?

23. 8 in. is what per cent of 12 in.? of 12 ft.? of 12 yd.?

24. A pint is what per cent of 1 qt.? of 1 gal.? of 6 gal.?

25. 56 ft. is 8% of what distance? 4% of what distance?

26. 7 ft. 6 in. is 10% of what distance? 8% of what distance? $33\frac{1}{3}\%$ of what distance?

27. 75 lb. is 25% less than what weight?

28. A bill of goods amounting to \$725 is allowed a discount of 15%. Find the net amount.

Exercise 50. Problems without Numbers

1. If you know the expenditures and the receipts for a year on a farm, how do you find the net profit or loss?

2. In Ex. 1 how do you find the average net profit or loss per acre?

3. If you know the dimensions of a rectangular field in rods, how do you find the area in square rods? in acres?

4. If you know the total annual cost of food for the cows on a farm and the number of cows, how do you find the average cost of food per cow?

5. If you know the average profit per cow on a farm and the number of cows, how do you find the total profit?

6. If you know the average per cent of butter fat in the milk from a certain herd of cows and the number of pounds of milk delivered at a creamery, how do you find the number of pounds of butter fat in this milk?

7. If a farmer has two cows, and knows the amount of milk furnished by each in a year and the per cent of butter fat in the milk of each cow, how does he find which cow produces the more butter fat?

8. If you know the weight of nitrogen in a ton of clover hay, how do you find the per cent of the nitrogen?

9. If you know the per cent of nitrogen in a ton of clover hay, how do you find the weight of the nitrogen?

10. If you know the per cent of nitrogen in clover hay, how do you find the amount of clover hay necessary to produce a given amount of nitrogen?

11. If you know the length of a rectangular field, how do you find the width that must be fenced off so as to inclose just an acre of land?

IV. ARITHMETIC OF INDUSTRY

Nature of the Work. We have thus far considered three important topics, the home, the store, and the farm, and have seen that each has its special kinds of problems. We shall now consider the problems of more general industries, such as manufacturing establishments of various kinds.

Teachers should draw problem material from local industries whenever possible. No textbook can do more than give a general survey of the subject, and it must always endeavor to present problems which are not too technical to be generally understood. In certain localities, however, where some single industry is prominent, more technical problems can safely be given because the students will naturally be familiar with the terms used.

In order to solve the problems which arise in the shop, it is necessary to review the operations with numbers. We shall therefore briefly review the operations with fractions.

Exercise 51. Fractions

1. Some plaster $\frac{1}{4}$ in. thick is coated with a finer plaster $\frac{3}{16}$ in. thick. How thick is the plaster then?

2. A plate of brass $\frac{1}{32}$ in. thick is laid on a plate of iron $\frac{3}{16}$ in. thick. What is the total thickness?

3. An iron rod of diameter $\frac{5}{8}$ in. is covered with a brass plating $\frac{1}{48}$ in. thick. What is now the diameter of the rod?

4. A table 4 ft. $4\frac{3}{4}$ in. long and 3 ft. $2\frac{1}{2}$ in. wide has a molding $\frac{1}{4}$ in. thick put around it. What is then the perimeter of the table?

5. A boy is making a dog kennel. One of the pieces of wood is 4 ft. 3 in. long, and from this he saws a piece 2 ft. $4\frac{1}{2}$ in. long. How long is the piece which remains?

6. A girl has a piece of ribbon $2\frac{1}{2}$ yd. long. She uses $14\frac{1}{2}$ in. for a hat. How much ribbon has she left?

7. From a board 14 ft. long a man saws off a piece 2 ft. $3\frac{1}{2}$ in. long and another piece 2 ft. $7\frac{3}{4}$ in. long. How long is the remaining part?

8. To a piece of cloth $4\frac{1}{2}$ yd. long another piece $8\frac{1}{2}$ in. long is sewed, and then $18\frac{1}{4}$ in. is cut off and used for making a bag. How many yards of cloth are left?

9. A plate of glass $18\frac{3}{4}$ in. by $23\frac{5}{8}$ in. was set in a picture frame that covered it $\frac{3}{8}$ in. from each edge. What are the dimensions of the glass not covered by the frame?

Perform the following additions:

10. $\frac{1}{4}+\frac{3}{4}+\frac{7}{8}$.

11. $\frac{3}{4}+\frac{3}{8}+\frac{5}{8}$.

12. $\frac{1}{4}+\frac{3}{4}+\frac{5}{8}$.

13. $\frac{3}{8}+\frac{7}{8}+\frac{3}{4}$.

14. $\frac{1}{2}+\frac{3}{4}+\frac{1}{12}$.

15. $\frac{1}{2}+\frac{3}{4}+\frac{5}{12}$.

16. $\frac{1}{2}+\frac{3}{8}+\frac{7}{16}$.

17. $\frac{3}{4}+\frac{5}{8}+\frac{9}{16}$.

18. $1\frac{2}{3}+\frac{2}{3}+3\frac{1}{12}$.

19. $2\frac{2}{3}+\frac{1}{2}+1\frac{1}{12}$.

20. $3\frac{2}{3}+2\frac{5}{6}+\frac{7}{12}$.

21. $3\frac{5}{8}+2\frac{1}{2}+\frac{11}{12}$.

22. In making a dress ruffle $4\frac{1}{8}$ in. wide when finished enough cloth must be allowed to turn in $\frac{5}{8}$ in. on one side and $\frac{7}{8}$ in. on the other. Find the width of cloth needed.

23. A gas fitter, in running a pipe into a schoolroom, has four pieces of pipe respectively 8 ft. $9\frac{1}{2}$ in., 6 ft. $2\frac{5}{8}$ in., 8 ft. $3\frac{1}{4}$ in., and 9 ft. 4 in. long, and finds he has 3 ft. 7 in. more than he needs. What is the length required?

Perform the following subtractions:

24. $5\frac{1}{2}-2\frac{3}{4}$.

25. $8\frac{1}{4}-3\frac{3}{8}$.

26. $6\frac{1}{3}-2\frac{2}{3}$.

27. $7\frac{5}{8}-4\frac{3}{4}$.

28. $7\frac{1}{2}-5\frac{3}{8}$.

29. $4\frac{1}{2}-2\frac{3}{4}$.

30. $6-4\frac{5}{16}$.

31. $5-2\frac{7}{12}$.

32. 8 in. $-2\frac{3}{4}$ in.

33. $9\frac{1}{2}$ in. $-6\frac{3}{32}$ in.

34. 8 in. $-1\frac{7}{32}$ in.

35. 9 in. $-3\frac{5}{24}$ in.

Division by a Fraction. We know that there are 3 thirds in 1, or that $1 \div \frac{1}{3} = 3$. From this we see that $6 \div \frac{1}{3} = 6 \times 3$.

That is, $6 \div \frac{1}{3} = 3 \times 6$,

and $6 \div \frac{2}{3} =$ half as much $= \dfrac{3 \times 6}{2} = 9$.

That is, $6 \div \frac{2}{3} = \frac{3}{2}$ of 6.

Therefore, *to divide by a common fraction, multiply by the reciprocal of that fraction.*

That is, $15 \div \dfrac{3}{5} = \dfrac{5}{\cancel{3}} \times \dfrac{\overset{5}{\cancel{15}}}{1} = 25.$

Similarly, $\dfrac{3}{8} \div \dfrac{5}{6} = \dfrac{\overset{3}{\cancel{6}}}{5} \times \dfrac{3}{\cancel{8}_{4}} = \dfrac{9}{20}.$

We shall now review both multiplication and division of fractions.

Exercise 52. Fractions

1. A tin cup is found to hold $\frac{15}{16}$ pt. When it is $\frac{4}{5}$ full the cup contains what part of a pint?

2. If a jar has a capacity of $\frac{15}{16}$ qt., what part of a quart will it contain when it is $\frac{2}{3}$ full? when it is $\frac{1}{2}$ full? What part full must it be to hold 1 pt.?

Perform the following operations:

3. $\frac{1}{2} \times \frac{3}{4}$.

4. $\frac{1}{2} \div \frac{3}{4}$.

5. $\frac{3}{4} \div \frac{1}{2}$.

6. $\frac{3}{4} \times \frac{3}{8}$.

7. $\frac{3}{4} \div \frac{3}{8}$.

8. $\frac{3}{8} \div \frac{3}{4}$.

9. $\frac{1}{2} \times \frac{5}{8}$.

10. $\frac{1}{2} \div \frac{5}{8}$.

11. $\frac{5}{8} \div \frac{1}{2}$.

12. $\frac{1}{2} \times \frac{1}{16}$.

13. $\frac{1}{2} \div \frac{1}{16}$.

14. $\frac{1}{16} \div \frac{1}{2}$.

15. $\frac{1}{4} \times \frac{7}{8}$.

16. $\frac{1}{4} \div \frac{7}{8}$.

17. $\frac{7}{8} \div \frac{1}{4}$.

18. $\frac{3}{4} \times \frac{7}{8}$.

19. $\frac{3}{4} \div \frac{7}{8}$.

20. $\frac{7}{8} \div \frac{3}{4}$.

21. $\frac{1}{4} \times \frac{1}{16}$.

22. $\frac{1}{4} \div \frac{1}{16}$.

23. $\frac{1}{16} \div \frac{1}{4}$.

24. $2 \times \frac{5}{16}$.

25. $2 \div \frac{5}{16}$.

26. $\frac{5}{16} \div 2$.

Exercise 53. The Pay Roll

1. The following is a week's pay roll of a manufacturer:

PAY ROLL		*For the week ending* Feb. 7, 1920									
No.	Name	No. of Hours per Day						Total Time	Wages per Hour	Total Wages	
		M.	T.	W.	T.	F.	S.				
1	M. S. Rowe	8	$7\frac{1}{2}$	8	$6\frac{1}{2}$	8	4	42	50	$21	
2	T. D. Bell	7	$7\frac{1}{2}$	8	8	8	4	$42\frac{1}{2}$	55	23	38
3	S. M. Lee	8	$7\frac{1}{4}$	8	$7\frac{1}{2}$	8	4	*	50	*	*
4	Louis Shea	8	7	6	8	$7\frac{1}{2}$	3	*	55	*	*
5	C. P. Grove	8	8	7	$7\frac{3}{4}$	8	$3\frac{1}{2}$	*	55	*	*
6	L. S. Cram	8	$6\frac{1}{2}$	8	0	7	$3\frac{3}{4}$	*	$27\frac{1}{2}$	*	*
	Totals	*	*	*	*	*	*	*		*	*

Fill each space marked with an asterisk (*).

Make out pay rolls for a week and insert names, when the men's numbers, the hours per day, and the wages per hour are as follows:

2. No. 1: $7\frac{1}{2}$, $7\frac{1}{2}$, $7\frac{1}{2}$, 8, 8, $3\frac{1}{2}$, 60¢; No. 2: 8, 8, 8, 8, 8, 4, 55¢; No. 3: 8, 8, 7, 7, 8, 4, 62¢.

3. No. 1: 7, 8, 8, 8, $7\frac{1}{2}$, 4, $62\frac{1}{2}$¢; No. 2: 8, 7, 8, 7, 6, 4, 60¢; No. 3: 8, $7\frac{1}{2}$, 8, 8, 8, 4, $48\frac{1}{4}$¢; No. 4: 8, 8, 8, 8, $7\frac{1}{2}$, 4, 65¢; No. 5: 8, 7, 8, 8, 8, 4, 72¢.

4. No. 1: 8, 7, 6, 8, 8, 4, $72\frac{1}{2}$¢; No. 2: 8, 7, 6, $8\frac{3}{4}$, $8\frac{1}{2}$, 4, 64¢; No. 3: 8, 6, 6, 8, 8, 4, 62¢; No. 4: 7, 8, $7\frac{3}{4}$, $6\frac{1}{2}$, 8, 4, 57¢; No. 5: 8, 6, 7, $7\frac{1}{2}$, $6\frac{1}{4}$, 4, 48¢.

5. No. 1: 7, 7, 8, 8, 8, 4, $62\frac{1}{2}$¢; No. 2: 8, 8, $7\frac{3}{4}$, $7\frac{1}{2}$, $7\frac{1}{4}$, 4, 65¢; No 3: 8, 8, 8, $7\frac{3}{4}$, $7\frac{3}{4}$, 4, 58¢; No. 4: $7\frac{1}{2}$, $7\frac{1}{4}$, $7\frac{1}{2}$, 8, $7\frac{3}{4}$, $3\frac{3}{4}$, 62¢; No. 5: 8, 8, 8, 8, 8, 4, $63\frac{1}{3}$¢.

6. Fill each space marked with an asterisk in the following pay roll, allowing double pay for all overtime:

PAY ROLL		*For the week ending* Jan. 10, 1920									
No.	Name	No. of Hours per Day						Total Time	Wages per Hour	Total Wages	
		M.	T.	W.	T.	F.	S.				
1	R. S. Jones	✓	$\overset{2}{\checkmark}$	✓	$1\frac{1}{2}\checkmark$	$1\frac{1}{2}\checkmark$	✓	54	40	*	*
2	M. L. King	$\overset{1}{\checkmark}$	✓	$\overset{1}{\checkmark}$	$1\frac{1}{2}\checkmark$	$6\frac{1}{2}$	$1\frac{1}{2}\checkmark$	$52\frac{1}{2}$	50	*	*
3	J. M. Mead	✓	$\overset{1}{\checkmark}$	$\overset{2}{\checkmark}$	$\frac{1}{2}\checkmark$	—	$1\frac{1}{2}\checkmark$	46	$62\frac{1}{2}$	*	*
	Totals	*	*	*	*	*	*	*		*	*

Before assigning Ex. 6 the teacher should explain that from one and a half to two times the regular hourly wage is usually paid for overtime, and that the check (✓) in the above pay roll means full time for the day. In this pay roll the full time is 8 hr. except on Saturday, when it is 4 hr. The symbol $\overset{2}{\checkmark}$ means 8 hr. + 2 hr. overtime. A dash (—) indicates absence. Part time, like $6\frac{1}{2}$ hr., is indicated as above on Friday for King. Since the allowance for overtime is double that for regular work, Jones's time is $8 + 8 + 8 + 8 + 8 + 4$ regular time and $2 \times (2 + 1\frac{1}{2} + 1\frac{1}{2})$ overtime, or 54 hr. in all. Explain the significance of the parentheses.

Make out pay rolls (inserting names) when the men's numbers, the hours per day, and the wages per hour are as follows, 8 hr. constituting a day's work except on Saturday, when it is 4 hr., and double pay being given for overtime:

7. No. 1: 8, 9, 8, 9, 8, 5, $67\frac{1}{2}$¢; No. 2: $8\frac{1}{2}$, 9, $9\frac{1}{2}$, 8, 8, 4, 65¢; No. 3: 8, 8, 8, 10, 8, 6, 62¢; No. 4: 8, 9, 9, 9, 8, 4, 60¢; No. 5: $8\frac{1}{2}$, $8\frac{1}{2}$, 9, 8, 8, 5, 60¢.

8. No. 1: 8, 10, 8, 10, 8, 6, 60¢; No. 2: $9\frac{1}{2}$, 8, 8, 9, 8, 4, 62¢; No. 3: 10, 10, 10, 10, 8, 5, $62\frac{1}{2}$¢; No. 4: 8, 8, 8, 8, 10, 9, $62\frac{1}{2}$¢; No. 5: 8, 8, 8, 9, $8\frac{1}{2}$, $6\frac{1}{2}$, 65¢.

Exercise 54. The Iron Industry

1. What is the weight of a steel girder that is 18′ 10″ long and weighs $46\frac{1}{4}$ lb. to the running foot?

2. What is the cost of 16′ 4″ of iron rod, $4\frac{1}{2}$ lb. to the foot, at $1\frac{7}{8}¢$ a pound?

3. The wooden pattern from which an iron casting is made weighs $6\frac{3}{4}\%$ as much as the iron. The pattern weighs $67\frac{1}{2}$ lb. How much does the casting weigh?

4. If steel rails weighing 180 lb. to the yard are used between New York and Chicago, a distance of 980 mi., how many tons of rails will be required for a double-track road between these cities?

5. An iron tire expands $1\frac{9}{16}\%$ on being heated for shrinking on a wheel. A certain wooden wheel needs a tire 16′ 8″ in circumference. How much longer will the tire be when thus heated?

6. If 3.5% of metal is lost in casting, how much metal must be melted to make a casting to weigh 77.2 lb.?

Since $100\% - 3.5\% = 96.5\%$, 77.2 is 96.5% of the weight.

7. In a certain blast furnace the casting machine turns out 40 pigs of iron per minute, averaging in weight 110 lb. each. If this machine runs for 312 da., 16 hr. a day, how many long tons (2240 lb.) of pig iron will it turn out?

8. Some years ago the average daily wages paid to employees in a certain mill was \$1.90, and the men worked 11 hr. a day, 6 da. in the week. At present the average daily wage is \$2.60 and the men work 8 hr. a day, 5 da. in the week and 5 hr. on Saturday. Considering the wages per hour, what has been the per cent of increase? Considering the hours per dollar of wages, what has been the per cent of decrease?

Exercise 55. Miscellaneous Problems

1. Sea Island cotton is usually shipped in bags of 150 lb., while Alabama cotton is shipped in bales of 500 lb. How many bags of Sea Island cotton at 28¢ a pound will equal in value 42 bales of Alabama cotton at 11¢ a pound?

2. The average number of wage earners engaged in the manufacture of cotton goods during a recent year was 379,366. The value of the materials was $431,602,540 and the value of the finished products was $676,569,335. What per cent of value was added by manufacture?

3. The United States produced 10,102,102 bales of cotton in the year 1900 and 11,068,173 bales in the year 1915. What was the per cent of increase?

4. The total value of the cotton raised in the United States in a recent year was $627,861,000, and the number of bales was 11,191,820. Find the average value of a bale.

5. During a recent year the United States produced 11,000,000 bales of cotton and used only 7,000,000 bales. The amount used in this country was what per cent of the total amount produced?

6. During a recent year 86,840 sq. mi. of cotton territory was invaded by the boll weevil. The total area infected at the end of the year was 409,014 sq. mi. What was the per cent of increase for the year?

7. In the days when cotton cloth was woven by hand an experienced weaver could turn out 45 yd. of cloth per week. At present a workman operating six power looms in a cotton mill will produce 1500 yd. per week. How long would it have taken the worker to do this with the hand loom? What is the per cent of increase in output per man with the power looms?

8. Before the invention of the cotton gin a laborer could separate in a day only $1\frac{1}{4}$ lb. of lint from the seed. At the present some gins turn out 10 bales of 500 lb. each per day. Such a machine does the work of how many men?

9. In making a silk lamp shade the following materials were used: $1\frac{1}{4}$ yd. silk @ \$1.10, $2\frac{1}{4}$ yd. silk fringe @ \$1.84, $\frac{3}{4}$ yd. silk net @ \$2.20, 1 frame costing 60¢. The labor and overhead charges amounted to \$3.25. The shade was marked \$14.50 but was sold at a discount of 10%. Find the gain per cent over the total cost.

Overhead charges, also called *overhead* or *burden,* means the general expense of doing business.

10. By repairing an automobile engine a mechanic increased its horse power $7\frac{1}{2}\%$ and reduced the amount of gasoline necessary to run it 3%. Before the repairs were made the engine developed 40 H.P. and used 2 gal. of gasoline on a 20-mile trip. How much gasoline per horse power did it use on a 50-mile trip after it was repaired?

The letters H.P. are commonly used for horse power.

11. It is desired to construct an engine that will generate 102.5 H.P. net, that is, actually available for use. It is found that 18% of the horse power generated is lost. This being the case, what horse power must be generated?

12. How many fleeces of wool averaging $6\frac{3}{4}$ lb. each must be used to make a bale of wool weighing 250 lb., and how many pounds will be left over?

13. If a wool sorter can sort 80 lb. of wool in a day, how many days will it take him to sort a shipment of 24 bales of 250 lb. each?

14. After scouring (cleaning) a shipment of 12,000 lb. of wool it weighed only 5240 lb. What per cent of the original weight was lost by scouring?

Exercise 56. Review Drill

1. Add 147.832, 29.68, 575, 0.387.

2. From 1000 subtract the sum of 148.9 and 9.368.

3. Multiply 78.4 by 9.86.

4. Divide 0.8 by 0.13 to three decimal places.

5. How much is a profit of $14\frac{1}{2}\%$ on a sale of cotton goods which cost \$1275.50?

6. Find the commission at $\frac{1}{8}\%$ on goods sold for \$15,000.

7. Goods listed at \$1450 are sold at a discount of 6%, 10%. Find the selling price.

8. How much is the profit, at $12\frac{1}{2}\%$ on cost plus overhead charges, on the sale of goods which cost \$1645.75, the overhead charges being \$268.50?

9. How much is the loss on a house which cost \$4500, including all charges, and which was sold at a loss of 6%?

10. Some goods which cost \$750, including all charges, were sold for \$675. What was the per cent of loss?

Write the results of the following:

11. 84 in. = (?) ft.
12. 84 oz. = (?) lb.
13. 84 ft. = (?) yd.
14. 84 pt. = (?) qt.
15. 84 qt. = (?) pt.
16. 2 sq. ft. = (?) sq. in.
17. 2 cu. ft. = (?) cu. in.
18. 2 sq. yd. = (?) sq. ft.
19. 72 in. = (?) ft.
20. 72 in. = (?) yd.

21. Make out an imaginary personal account of six items on each side, and balance the account.

22. Write a bill of \$45 for rent due; a check to pay half this rent; a receipt for this part of the rent; a note for the rest of the rent.

Exercise 57. Problems without Numbers

1. If you know the wages per hour and the number of hours worked each day by each man, without overtime, how do you find the total wages due all the men in a shop in a week?

2. If you know the regular wages due a man per hour, the wages for overtime, and the number of hours he works each day in a week, some of these being overtime, how do you find his total wages for a week?

3. If you know the weight of a steel girder per running foot and the length of the girder, how do you find the total weight?

4. If you know the weight of a girder and its length, how do you find its weight per running foot?

5. If you know the weight of each of several bales of cotton and the price paid per pound, how do you find the gain or loss to a firm that buys this cotton on a basis of 500 lb. to the bale?

6. If you know the shipping rate per hundredweight from where you live to Liverpool, and know the weight of a shipment, how do you find the total charge for freight?

7. If a farmer has wheat to ship to Chicago and knows the freight rate per bushel or carload and the number of bushels or carloads, how does he find the freight charges?

8. If a farmer knows the charges for shipping a certain number of bushels of corn to Chicago, how does he find the freight rate per bushel?

9. If you know the average weight of a fleece of wool and the number of fleeces a sheep grower has, how do you find the number of bales of wool weighing 250 lb. each and the amount, if any, that will be left over?

V. ARITHMETIC OF THE BANK

Saving. A boy who puts 1¢ each day into a toy bank will have enough in six months to buy a catcher's mitt or three baseball bats. A girl who saves 10¢ a day will have enough in a month or two to buy a good pair of shoes. A man who saves $1 a day will have enough in a few years to buy a building lot in some place where he may care to live. We often get the things that we need for comfortable living or the things that give us legitimate pleasure, by saving a little at a time.

The following suggestion to teachers will be found helpful:

Begin with a brief discussion of the need and value of saving money. There are many of us who never learn how to save money wisely. Many of us prefer to gratify our immediate desires rather than to provide for the future. Why is this a bad plan? On the other hand, there are others of us who, in order to save for the future, deny ourselves the things which it would be real economy to buy. There is always the temptation to live extravagantly. Extravagance includes not only living beyond our means but also spending money foolishly. Every boy and every girl should begin early in life to form the habit of saving, no matter if it be but a few dollars a year.

Exercise 58. Saving

1. A man wishes to buy an automobile that costs $780. If he saves $2 every week day, in how many weeks will he save enough to buy the car?

2. A boy wishes to buy a camera that costs $6.60. If he saves 10¢ every week day, in how many weeks will he save enough to buy the camera?

3. A girl wishes to buy a purse that costs 60¢. If she saves 5¢ every week day, in how many weeks will she save enough to buy the purse?

4. A man who has been smoking six cigars a day, which he buys at the rate of three for a quarter, decides to give up smoking and save the money. How much will this saving amount to in 5 yr.?

In all such problems the year is to be considered as 365 da. (or 313 da., excluding Sundays), although in 5 yr. there will probably be one leap year and there may be two leap years, and although a year need not have exactly 52 Sundays.

5. A boy earns some money by selling papers. He finds that he can easily save 15¢ a day, excluding Sundays. If he does this for 5 yr., how much will he save in all?

6. A woman in a city has a telephone for which she is charged 5¢ for each call. She finds that she can economize by reducing the number of her telephone calls on an average four a day, including Sundays. If she does this for 3 yr., how much will she save?

Make out accounts, inserting dates, items of receipts and payments, and the balances, given the following:

7. On hand, \$4.20. Receipts: 20¢, 40¢, 25¢, \$1, 30¢, 65¢. Payments: 25¢, 30¢, 10¢, 75¢.

8. On hand, \$4.30. Receipts: 10¢, 15¢, 60¢, 55¢. Payments: 30¢, 42¢, 75¢, 5¢, 30¢, 60¢, 20¢, 5¢.

9. On hand, \$4.63. Receipts: 10¢, 12¢, 32¢, 12¢, \$1.10. Payments: 25¢, 35¢, 5¢, 24¢, \$1.50, 25¢, 50¢.

10. On hand, \$5.10. Receipts: \$1.25, 35¢, 75¢, 42¢, 68¢, 70¢, \$1.22. Payments: \$1.50, 70¢, 5¢, 10¢.

11. On hand, \$1.30. Receipts: \$1, 40¢, 5¢, 15¢, 10¢, 25¢, 12¢, 16¢, 10¢. Payments: 10¢, 20¢, 5¢, 5¢, 22¢, 3¢.

12. On hand, \$3.20. Receipts: 70¢, 60¢, 10¢, 3¢, 15¢, 10¢, 25¢, 5¢, 5¢, 30¢, 52¢, 30¢. Payments: \$1.50, 50¢, 90¢, 70¢, 18¢, 12¢, 30¢, 45¢, 25¢.

Bank Account Essential. One thing that is essential at some time to everyone who hopes to succeed is a bank account. A reliable person may "open an account," as it is called, as soon as he begins to save even small amounts.

People who are saving money usually keep it in a bank until they have enough for investing permanently. Certain kinds of banks, such as savings banks and trust companies, not only guarantee to take care of all money left with them by depositors but also pay a certain per cent of interest. National banks also generally allow interest on what are called *inactive* accounts; that is, deposits that remain undisturbed for some time.

Many schools have found it interesting and profitable to organize school banks, electing the officers and carrying on a regular banking business, either with small amounts of real money placed on deposit by students and transferred by the teacher to some bank or trust company, or with imitation money. Such exercises should not, however, interfere with the work in computing.

Savings Bank. To deposit money a person goes to a bank, says that he wishes to open an account, and leaves his money with the officer in charge. The officer gives him a book in which is written the amount deposited, and the depositor writes his name in a book or on a card, for identification. When he wishes to draw out money, he takes his book to the bank, signs a receipt or a check for the amount he desires, and receives the money, the amount being entered in his book.

Students should be told of the advantages of opening even small accounts at a savings bank. A boy who deposits \$1 a week for 10 yr. in a bank paying 2% every 6 mo. and adding it to the account, will have \$631.54 in 10 yr., and a man who puts in \$10 a week will have about ten times as much, or \$6317.16.

The class should be told about trust companies, which take charge of funds, manage estates, and pay interest on deposits.

Exercise 59. Saving

1. How much will 25¢ saved each working day, 310 such days to the year, amount to in 10 yr.?

2. If a boy, beginning at the age of 14 yr., saves 25¢ a day for 310 da. a year and deposits it in a bank, how much has he when he is 21 yr. old, not counting interest?

Interest and withdrawals are not to be considered in such cases.

3. If a man saves $3.25 a week out of his wages and continues to do this 52 wk. in a year for 12 yr., how much money will he save?

4. If a father gives his daughter on each birthday until and including the day she is 25 yr. old as many dollars as she is years old, depositing it for her in a savings bank, how much has she when she is 25 yr. old?

5. A merchant saves $750 the first year he is in business. The second year he saves one third more than in the first year. The third year his savings are only 85% as much as the second year. The fourth year they increase 30% over the third year. How much does he save in the four years?

6. A man works on a salary of $18 a week for 52 wk. in a year. His expenses are $3.75 a week for house rent, 60% as much for clothing, 300% as much for food as for clothing, and 20% as much for other necessary expenses as for food. How much of his salary can he deposit each year in the savings bank?

7. A clerk had a salary of $12 a week two years ago and a commission of 2% on goods he sold. That year he worked 50 wk. and sold $4800 worth of goods. Last year his salary was increased 25%, his rate of commission remaining the same. He worked 48 wk. and sold goods to the amount of $5600. How much was his income increased?

Interest. If Mr. James has a house and lot worth \$5000, and rents it to Mr. Jacobs at \$42 a month, his income from the rent is $12 \times \$42$, or \$504 a year. This is a little more than 10% of the value of the property, but out of it Mr. James has to pay for various expenses, such as insurance, repairs, and taxes.

If Mr. James has \$5000 and lends it to Mr. Jacobs at the rate of 6% a year, his income from this transaction is 6% of \$5000, or \$300 a year.

Money paid for the use of money is called *interest.* In the above illustration about the lending of money \$300 is the interest for 1 yr., 6% is the *rate* of interest, and \$5000 is the *principal.*

Schools do not require, as formerly, the learning of many definitions. What is necessary is that the student should use intelligently such terms as interest, rate, and principal.

Men often have to borrow money to carry on their business. For example, a merchant may wish to buy a lot of holiday goods, feeling sure that he can sell them at a profit. In this case it is good business for him to borrow the money, say in November, taking advantage of all cash discounts allowed, and then to repay the money in January after the goods are sold. If, for example, he needs \$1000 for 2 mo. and can borrow it from a bank at the rate of 6% a year, he will have to pay $\frac{2}{12}$ of 6% of \$1000, or 1% of \$1000, or \$10, a sum which he can easily afford to pay for the use of the money.

To find the interest on any sum of money for part of a year first find the interest for 1 yr. and then find it for the given part of a year.

Formal rules for such work need not be memorized. An example or two may profitably be worked on the blackboard before studying the next page.

Exercise 60. Interest

Examples 1 to 27, oral

Find the interest on the following amounts for 1 yr. at the given rates:

1. \$1000, 5%.	**6.** \$150, 4%.	**11.** \$400, 6%.
2. \$1000, 6%.	**7.** \$250, 4%.	**12.** \$400, $5\frac{1}{2}$%.
3. \$1000, $4\frac{1}{2}$%.	**8.** \$500, 5%.	**13.** \$1000, $3\frac{1}{2}$%.
4. \$2000, 5%.	**9.** \$600, 6%.	**14.** \$1000, $4\frac{1}{4}$%.
5. \$3000, 6%.	**10.** \$800, $3\frac{1}{2}$%.	**15.** \$1000, $4\frac{3}{4}$%.

Find the interest on the following amounts for 6 mo. at the given rates:

16. \$100, 6%.	**18.** \$1000, 5%.	**20.** \$3000, 4%.
17. \$300, 6%.	**19.** \$2000, 5%.	**21.** \$1000, $3\frac{1}{2}$%.

Find the interest on the following amounts for 1 yr. 6 mo. at the given rates:

22. \$1000, 4%.	**24.** \$2000, 6%.	**26.** \$5000, 4%.
23. \$1000, 5%.	**25.** \$5000, 5%.	**27.** \$2500, 4%.

28. A man having \$17,250 invested in business has found that his net profits average 16% a year on the investment. He is offered \$25,000 for the business, and he could invest the money at $4\frac{1}{2}$%. If he sells out and retires, what is his annual loss in income?

29. In April a coal dealer borrowed \$66,420 at 5%. With this he purchased his summer's supply of coal at \$5.40 a ton, his overhead charges being 30¢ a ton. He sold the coal at \$6.68 a ton, the buyers paying for the unloading and delivery, and he paid his debt in October after keeping the money 6 mo. How much did he gain?

Interest for Months and Days. Suppose that a man borrows from a bank \$400 on Sept. 10, 1919, at 6%. What will the interest amount to Aug. 7, 1920?

yr.	mo.	da.	
1920	8	7	= second date
1919	9	10	= first date
	10	27	= difference in time

Taking, as is usual, 30 da. to the month, the difference in time is 327 da. We therefore have

$$\frac{327}{360} \text{ of } 6\% \text{ of } \$400 = \frac{\overset{109}{\cancel{327}} \times \cancel{6} \times \overset{4}{\cancel{\$400}}}{\underset{90}{\cancel{360}} \times \underset{15}{\cancel{100}}} = \$\frac{109}{5} = \$21.80.$$

Hence the interest due August 7, 1920, is \$21.80.

Banks usually lend money for a definite number of days or else require payment to be made on demand. In either case they compute the interest for the days that the borrower has the money and not for months and days. To enable them to compute the interest easily they have interest tables. Private individuals, however, occasionally have to compute interest for months and days, and in that case they may proceed as in the above problem.

It is a waste of time for the student to find the interest on very small or very large sums of money, for very short or very long periods, or at more than legal rates. A few such examples may be given, however, for practice in computation. In general, interest is now reckoned on such a sum as \$750 rather than \$749.75, and for periods not exceeding 90 da. rather than one involving years, months, and days. Teachers should advise the students that if the interest is for more than 1 yr. they should first find it for the given number of years, and then, by the above method, for parts of a year. Such cases are, however, rapidly becoming obsolete. Banking facilities make it rare to find interest periods for years, months, and days.

Interest for 30, 60, and 90 Days. In borrowing money at a bank the time for which the money is borrowed is usually 30 da., 60 da., or 90 da., except when repayment is to be made on demand. Since 6% is the most common rate, it is convenient to be able to work mentally the common types of interest examples.

How much interest must you pay if you borrow \$500 from a bank at 6% for 60 da.? for 30 da.? for 90 da.?

Since 60 da. $= \frac{60}{360}$ yr. $= \frac{1}{6}$ yr., the interest on \$500 for 60 da. is $\frac{1}{6}$ of 6% of \$500, or 1% of \$500, or \$5.

For 30 da. the interest is $\frac{1}{2}$ of 1% of \$500, or \$2.50.
For 90 da. the interest is $\frac{3}{2}$ of 1% of \$500, or \$7.50.

From this work state a simple rule for finding the interest at 6% for 60 da.? for 30 da.? for 90 da.?

Exercise 61. Interest

Examples 1 to 15, oral

Find the interest at 6% on the following amounts:

1. \$400, for 60 da.
2. \$650, for 30 da.
3. \$725, for 60 da.
4. \$875, for 60 da.
5. \$900, for 90 da.
6. \$840, for 60 da.
7. \$350, for 90 da.
8. \$450, for 30 da.
9. \$950, for 30 da.
10. \$860, for 30 da.

11. Find the interest on \$600 for 60 da. at 5%.

The interest at 6% is \$6, and so at 5% it is $\frac{5}{6}$ of \$6.

12. Find the interest on \$3000 for 30 da. at 5%.
13. Find the interest on \$240 for 90 da. at 5%.
14. Find the interest on \$600 for 60 da. at 4%.
15. Find the interest on \$1200 for 30 da. at 3%.

16. Find the interest on \$400 for 2 yr. 10 mo. 27 da. at 6%.

The interest for 2 yr. is 2 × 6% of \$400, or \$48, and for 10 mo. 27 da. is \$21.80, as found on page 83. Hence the total interest is \$69.80.

As already stated, such examples are becoming more rare. A few are given on this page, chiefly as exercises in computation.

Find the interest on the following:

17. \$1250 for 2 mo. 17 da. at 5%.

18. \$1500 for 7 mo. 23 da. at 6%; at $5\frac{1}{2}$%.

19. \$2400 for 8 mo. 11 da. at 5%; at 6%; at $5\frac{1}{2}$%.

20. \$575 for 2 yr. 9 mo. 15 da. at 5%; at $5\frac{1}{2}$%.

21. \$850 for 3 yr. 10 mo. 6 da. at $5\frac{1}{2}$%; at 6%.

22. \$925 for 4 yr. 10 mo. 6 da. at 6%; at 5%.

23. A dealer bought 24 sets of furniture on Nov. 1, at \$50 a set, promising to pay for them later, with interest at 6%. He paid the bill on the following Jan. 16. What was the amount of principal and interest?

24. A man borrowed \$750 on Mar. 10, at 6%, and \$1600 on Apr. 10, at 5%. He paid the entire debt on July 10 of the same year. How much did he pay in all?

25. A man borrowed \$750 on May 1, at 5%, and \$1800 on July 5, at $4\frac{1}{2}$%. He paid both debts with interest on Dec. 16 of the same year. How much did he pay in all?

26. What is the total amount of principal and interest on \$950 borrowed Mar. 10, at 6%, and \$1600 borrowed May 15, at 5%, the payment in both cases being made on Oct. 20 of the same year?

27. A man borrowed \$750 on May 9, at 5%, and \$625 on June 15, at 6%, each loan to run for 60 da. When was each due, and how much was the total interest?

Interest at Savings Banks. Savings banks usually pay interest every six months or every three months. This interest is added to the principal, and the total amount then draws interest.

Compound Interest. When interest as it becomes due is added to the principal and the total amount then draws interest, the investor is said to receive *compound interest* on his money.

Compound interest is not commonly used, but if one collects interest when due and at once reinvests it, he practically has the advantage of compound interest. The method of finding compound interest is substantially the same as that used in simple interest.

For example, how much is the amount of \$2000 in 2 yr., deposited in a savings bank that pays 4% annually, the interest being compounded semiannually? How much is the compound interest?

\$2000.	= first principal
.02	
\$40.	= int. first 6 mo.
2000.	
\$2040.	= amt. after 6 mo.
.02	
\$40.80	= int. second 6 mo.
2040.	
\$2080.80	= amt. after 1 yr.
.02	
\$41.62	= int. third 6 mo.
2080.80	
\$2122.42	= amt. after $1\frac{1}{2}$ yr.
.02	
\$42.45	= int. fourth 6 mo.
2122.42	
\$2164.87	= amt. after 2 yr.
2000.	
\$164.87	= int. for 2 yr.

Simple interest for the same time is \$160, or \$4.87 less than the compound interest.

Here the compound interest has been found exactly, but savings banks pay interest only on the dollars and not on the cents.

Savings Bank Account. The following is a specimen account at a savings bank which pays interest at the rate of 4% a year, the interest being payable semiannually, on January 1 and July 1, on the smallest balance on deposit at any time during the previous interest period:

Date		Deposits		Interest		Payments		Balance	
1922									
July	1	600	50					600	50
July	20	75						675	50
Sept.	6					120		555	50
Dec.	7	60						615	50
Dec.	20					65		550	50
1918									
Jan.	1			11				561	50
May	9	200						761	50
July	1			11	22			772	72

The smallest balance during the first interest period is $550.50. Interest is computed on the dollars only, the cents being neglected. At 4% per year the interest for 6 mo. on $550 is 2% of $550, or $11. In the second period the smallest balance is $561.50, and therefore the interest is 2% of $561, or $11.22.

Some banks allow interest from the first of each month; others from the first of each quarter; others, as above, from the first of each half year. The interest is computed on the smallest balance on hand between this day and the next interest day, and is usually added every half year, although it is sometimes added every quarter.

Students should ascertain the local custom as to savings banks.

Exercise 62. Compound Interest

Find the amount of principal and interest at simple interest, and also at interest compounded in a savings bank annually:

1. \$3000, 2 yr., 5%.
2. \$3000, 4 yr., 6%.
3. \$2000, 4 yr., 4%.
4. \$3250, 4 yr., 3%.
5. \$3750, 4 yr., 3%.
6. \$2750, 4 yr., $3\frac{1}{2}$%.
7. \$825.50, 5 yr., $3\frac{1}{2}$%.
8. \$2000, 6 yr., 4%.
9. \$625.50, 4 yr., $4\frac{1}{2}$%.
10. \$875.50, 3 yr., $4\frac{1}{2}$%.

Find the amount of principal and interest, the interest being compounded in a savings bank semiannually:

11. \$400, 3 yr., 4%.
12. \$600, 2 yr., 4%.
13. \$850, 2 yr., 6%.
14. \$900, 3 yr., 3%.
15. \$900, 3 yr., 4%.
16. \$600, 2 yr., 4%.
17. \$2000, 2 yr., $4\frac{1}{2}$%.
18. \$2000, 3 yr., 4%.
19. \$3000, 2 yr., $3\frac{1}{2}$%.
20. \$3000, 4 yr., 4%.

21. If you deposited \$140 in a savings bank on July 17, 1919, and \$35 on Feb. 9, 1920, and if you have made no withdrawals, to how much interest are you entitled July 1, 1920? In this bank on July 1 and Jan. 1 interest on each deposit at 4% per year is credited from the day of deposit if on the first day of a month, and otherwise from the first day of the following month.

22. If a man deposits \$1500 in a savings bank on Jan. 1, \$215 on Feb. 1, \$140 on May 7, \$270 on Sept. 11, and \$243 on Dec. 3, and makes no withdrawals, how much will he have to his credit on the following Jan. 1? In this bank on July 1 and Jan. 1 interest on each deposit at 4% per year is credited from the day of deposit if on the first day of a month, and otherwise from the first day of the following month.

Postal Savings Bank. The United States government conducts a savings bank in connection with the post office. Although all savings banks are carefully regulated and inspected by the state governments, there are many persons who are willing to take the smaller rate of income which the postal savings bank pays, because of the fact that our government guarantees the payment of their money.

Any person of the age of 10 yr. or over may deposit money in amounts of not less than \$1, but no fractions of a dollar are accepted for deposit. No one can deposit more than \$100 in any one calendar month or have a balance at any time of more than \$500, exclusive of accumulated interest. Deposits may be made at the larger post offices, and a depositor receives a postal savings certificate for the amount of each deposit. Interest is paid by the government at the rate of 2% for each full year that the money remains on deposit, beginning on the first day of the month next following the one in which the deposit is made. Interest is not paid for any fraction of a year. A person may exchange his deposits in sums of \$20 or multiples of \$20 for bonds bearing interest at $2\frac{1}{2}\%$.

Exercise 63. Postal Savings Bank

All work oral

Find the interest for 1 yr. on the following deposits:

1. \$30. **2.** \$40. **3.** \$75. **4.** \$300. **5.** \$500.

Find the interest for 2 yr. on the following deposits:

6. \$50. **7.** \$60. **8.** \$100. **9.** \$200. **10.** \$500.

Find the interest for 1 yr. on a $2\frac{1}{2}\%$ bond of:

11. \$80. **12.** \$240. **13.** \$360. **14.** \$480. **15.** \$500.

Bank of Deposit. When a man has money enough ahead to pay his bills by checks, he will find it convenient to have an account with a bank such as merchants commonly use, sometimes called a *bank of deposit.*

Such banks do not pay interest on small accounts, the deposit being a matter of convenience and safety. If a man wishes to open an account he sometimes has to give references, for banks do not wish to do business with unreliable persons. A man's credit in business is always a valuable asset.

In some sections of the country banks receive deposits under two classes of accounts, savings accounts and checking accounts. In the former case they act as savings banks; in the latter, as banks of deposit. For the purposes of the school it is not necessary to consider this difference further. Students should, however, investigate the local custom in the matter.

Deposit Slip. A man, when he deposits money or checks in a bank, fills out a *deposit slip* similar to the one here shown.

DEPOSITED FOR CREDIT OF

. . . . . .

IN THE

SECOND NATIONAL BANK

OF THE CITY OF NEW YORK

_____ 191____

	DOLLARS	CENTS
BILLS_____		
COIN_____		
CHECK ON_____B'K		
" _____ "		
" _____ "		

Sometimes the depositor enters the name of the bank on which each check is drawn; sometimes the receiving teller at the bank does this by writing the bank's number; and sometimes it is not entered at all. These are technicalities that do not concern the school.

Exercise 64. Deposit Slips

Write or fill out deposit slips for the following deposits, inserting the name of the depositor and of the bank:

1. Bills, \$375; silver, \$60; check on Garfield Bank, \$87.50; check on Miners Bank, \$627.75.

2. Bills, \$423; gold, \$175; silver, \$235.75; check on Corn Exchange Bank, \$736.90.

3. Bills, \$135; check on Second National Bank of New York, \$425; check on Chase National Bank, \$75.40.

4. Bills, \$1726; gold, \$100; silver, \$200; check on Merchants Bank, \$245.50; check on Union Bank, \$275.40.

5. Bills, \$1275; checks on Harriman National Bank, \$146.50, \$200; checks on Jefferson Bank, \$325, \$86.50.

6. Gold, \$100; checks on First National Bank, \$175, \$240, \$32.80; checks on Sherman Bank, \$37.42, \$61.85.

7. Bills, \$2475; silver, \$275.50; check on Case Bank, \$43.50; check on Miners National Bank, \$250.

8. Bills, \$345; silver, \$350.75; gold, \$480; check on Merchants National Bank, \$455; check on Farmers Trust Co., \$262.50; check on City Bank, \$1000.

9. A man deposited \$475.75 in cash to-day, a check for 50% of a debt of \$675 due him, and a check in payment for 45 yd. of velvet at \$2.25 a yard less $33\frac{1}{3}\%$ discount. Make out a deposit slip.

10. A merchant received cash for 8 doz. forks @ \$14.75, $5\frac{1}{2}$ doz. teaspoons @ \$13, a watch costing \$40.50, and 4 clocks @ \$7.75. He also received a check on the Lincoln Trust Co. for 3 doz. dessert spoons @ \$17.75 and $4\frac{1}{2}$ doz. nutcrackers @ \$9. He deposited all this in a bank. Make out a deposit slip.

Check. *A check book* containing *checks* and *stubs*, substantially as follows, although often varying in certain details, is given the depositor when he opens an account.

No. 896 New York, May 21, 1920

Second National Bank

Pay to the order of Myron P. Jones $82.75

Eighty-two 75/100 Dollars

James K. Rogers

CHECK

No. 896
Date May 21, 1920
To M. P. Jones
For Groceries
Amt. $82.75

STUB

The person to whom a check is payable is called the *payee*. In the above example Myron P. Jones is the payee. A check may be made payable to "Self," in which case the drawer alone can collect it; or to the order of the payee, as in the above check, in which case the payee must *indorse* it, that is, he must write his name across the back; or to the payee or "bearer," or to "Cash," in which cases anyone can collect it.

The *indorsement* made by Mr. Jones would appear on the back in the form here shown:

Myron P. Jones

The teacher should explain to the class the nature of checks, the different ways of filling them out, and the end on which they should be indorsed. The teacher should explain the advantages of the various methods of making the checks payable and the students should write or fill out various styles of checks.

Exercise 65. Bank Deposits

1. If your deposits in a bank have been \$58.65, \$43, \$25, \$80, \$95, \$25.75, \$12.50, and \$9.50, and you have drawn checks for \$8.25, \$16.30, \$15.75, \$16.48, and \$25, what is then your balance at the bank?

2. A man earning \$22.50 a week deposits \$15 every Saturday, and each Monday gives a check for \$4.50 for his board. What will be his balance in 13 wk.?

3. If your deposits in a bank have been \$68.45, \$92.30, \$47.60, \$38.50, \$78.75, and \$96.70, and you have drawn checks for \$8.55, \$23.65, \$8.58, \$48.75, and \$34.60, what is your balance?

4. A merchant having \$980.75 in the bank deposits during the next week \$185.50, \$97.85, \$135.50, \$86.85, and \$236.80. He gives checks for \$89.65, \$37.20, \$93.60, \$15.20, \$248.70, and \$39.80. What is now his balance?

5. A man having \$825.60 in the bank gives a check for \$128.75. He then deposits checks for \$75.80, \$126.75, \$234.80, and \$42.80. During this time he gives checks for \$125.80 and \$24.75. What is now his balance?

6. A merchant having \$828.50 in the bank deposits \$567.80, \$245.50, \$89.65, \$482.86, \$429.50, and \$376.50, and draws checks for \$427.50, \$38.95, \$67.82, \$568.70, and \$122.58. He also pays by check a bill for \$125.40 less 10%, another bill for \$86 less 4%, and another for \$48.75 less 6%. What is now his balance?

7. A merchant having \$1026.92 in the bank deposits \$488.75, \$928.75, \$386.48, \$442.80, \$196.85, \$327.75, and draws checks for \$96.75, \$286.75, \$342.80, \$438.50. He also gives a check for \$230 plus interest for 4 mo. at 5%. What is now his balance?

Promissory Note. A paper signed by a borrower, agreeing to repay a specified sum of money on demand or at a specified time, is called a *promissory note*, or simply a *note*.

The sum borrowed is called the *principal*, or, if a note is given, the *face of the note.*

The sum of the principal (or face) and the interest is called the *amount* of the note.

A note should state the date, face, rate, person to whom payable, and time to run (time before it is due to be paid), and that it has been given for value received by the maker. The following is a common form for a time note:

$75.00 NEW YORK, February 7, 1921

Six months after date, for value received, I promise to pay to John Johnson or order, Seventy-five 00/100 Dollars, with interest at 5%.

Frank Francis

The following is a common form for a demand note:

$50.00 NEW YORK, May 2, 1921

On demand, for value received, I promise to pay to Robert R. Jones or order, Fifty 00/100 Dollars, with interest at 6%.

James P. Poole

Parties to a Note. The person named in a note as the one to whom it is payable is called the *payee*. The person who signs a note is called the *maker*.

Indorsing a Note. If the payee sells the note, he must, when it is payable to himself or order, *indorse* it.

A note is indorsed by the payee by writing his name across the back. The indorser must pay the note if the maker does not.

A note payable to John Johnson or *bearer* may be sold without indorsement. Such notes are not common.

If the payee wishes to sell the note without being responsible for the payment in case the maker should fail to pay it, he may write the words "without recourse" across the back, and write his name underneath. This means that he relinquishes all title to it and that the buyer cannot come back (have recourse) on him. The following are the forms:

Indorsement in Blank	Indorsement in Full	Limited Indorsement
John Johnson	Pay to the order of John Roberts John Johnson	Without recourse John Johnson

Teachers should explain fully the meaning of these several indorsements, and should have the students indorse notes properly.

Rate of Interest. The United States borrows money at rates of about 3% to $3\frac{1}{2}$%. Savings banks pay depositors about 3% or 4%. In cities, on good security, borrowers usually pay from 4% to 6%.

When a note bears interest, but the rate is not specified, it bears interest at a certain rate fixed by the law of the state. In many states this rate is 6%.

In most states, if a note falls due on a Sunday or a legal holiday, it is payable on the next business day.

Exercise 66. Promissory Notes

1. Compute the amount of the first note on page 94.

2. Write a promissory note, signed by A and payable to B, for \$75, due in 1 yr., at 6%. Find the amount.

3. F. H. Ryder borrows \$750, at 6%, for 1 yr., from M. P. Read. He gives a note payable to Mr. Read or order. Mr. Read sells the note to F. N. Cole. Make out the note, indorse it in full, and find the amount.

4. Make out a note like the one referred to in Ex. 3, but for \$725. Indorse it in blank and find the amount due at the end of the year. Write a check for this amount.

5. Make out a note like the one referred to in Ex. 3, but for \$1250. Indorse it without recourse and find the amount due at the end of the year. Write a check for this amount.

6. Write a note for \$275, bearing interest at 6% and payable in 6 mo. Insert names and dates, and indorse it payable to the order of John Ball, with a second indorsement by which Mr. Ball transfers it to James Clay.

Make out and indorse, payable to the order of the buyer, the following notes, and find the amount due on each:

	Maker	Payee	Buyer	Face	Rate	Time
7.	A. N. Cole	P. R. Carr	J. R. Hall	\$775	6%	4 mo.
8.	A. R. Doe	E. F. Dun	M. L. King	\$650	5%	8 mo.
9.	D. E. Bell	E. L. Cree	A. K. Iles	\$550	6%	3 yr.
10.	O. N. Olds	B. R. Hall	B. S. Hill	\$350	5%	5 mo.
11.	S. M. Roe	C. N. King	O. M. Coe	\$225	6%	2 yr.
12.	A. J. Burr	G. F. Ray	A. R. Carr	\$850	$5\frac{1}{2}$%	6 mo.
13.	A. B. Bain	G. F. Dow	F. E. True	\$950	$4\frac{1}{2}$%	9 mo.

Bank Discount. When a man borrows from a bank on a time note he pays the interest in advance. Interest is not mentioned in the note, because it has already been paid.

Interest paid in advance on a note is called *discount.*

Teachers should call the attention of the students to the fact that the same word is used for bank and commercial (trade) discount, explaining that the mathematical process is the same in both cases; that is, finding some per cent of a number.

Unless otherwise directed, always call 30 da. a month.

Proceeds. The face of a note less the discount is called the *proceeds.*

What are the discount and proceeds of a note for \$225 for 6 mo. at 5% ?

The discount (interest) for 1 yr. is 5% of \$225, or \$11.25.
The discount for 6 mo. is $\frac{1}{2}$ of \$11.25, or \$5.63.
The proceeds are \$225 − \$5.63, or \$219.37.

Exercise 67. Bank Discount

Find the discounts and the proceeds on the following:

1. \$300, 1 mo., 6%.
2. \$500, 30 da., 6%.
3. \$750, 2 mo., 5%.
4. \$475, 3 mo., 6%.
5. \$825, 2 mo., 5%.
6. \$500, 90 da., 6%.
7. \$475, 6 mo., 6%.
8. \$800, 90 da., 6%.
9. \$150, 45 da., 6%.
10. \$600, 1 mo., $5\frac{1}{2}$%.
11. \$300, 60 da., 5%.
12. \$575, 2 mo., $4\frac{1}{2}$%.
13. \$400, 4 mo., $5\frac{1}{4}$%.
14. \$800, 2 mo., $3\frac{1}{2}$%.
15. \$5000, 63 da., 5%.
16. \$3350, 93 da., 6%.
17. \$1250, 10 da., 6%.
18. \$2500, 15 da., 6%.
19. \$1500, 20 da., 6%.
20. \$1250, 45 da., 5%.

Find the discounts and the proceeds on the following:

21. \$675, 30 da., 6%.
22. \$750, 90 da., 5%.
23. \$850, 30 da., 6%.
24. \$3500, 60 da., 5%.
25. \$4250, 60 da., 6%.
26. \$4500, 90 da., 5%.
27. \$3000, 90 da., 5%.
28. \$4500, 90 da., $5\frac{1}{2}$%.
29. \$3750, 30 da., $4\frac{1}{2}$%.
30. \$136.75, 30 da., 6%.
31. \$275.50, 60 da., 5%.
32. \$42,000, 30 da., 5%.

33. Make out a 60-day note for \$450, dated to-day, payable to R. D. Cole's order at some bank. Discount it at 6%.

34. Make out a 30-day note for \$350, dated to-day, payable to Frank Lee's order at some bank. Discount it at 5%.

35. Make out a 60-day note for \$960, dated to-day, payable to Ray Lang's order at some bank. Discount it at 5%.

36. Make out a 90-day note for \$3000, dated to-day, payable to L. D. Baldwin's order at some bank of which you know. Discount it at 6%.

37. A man's bank account shows deposits of \$175.50, \$68.50, \$50, \$300, \$40, \$75, \$100, \$125, and \$500; checks drawn, \$43.75, \$125.50, \$62, \$5, and \$125.35. He needs \$4500 to start him in business and wishes to keep about \$500 in the bank. How much money, to the nearest \$100, should he borrow?

38. If the man in Ex. 37 makes out a note for this amount for 90 da. at 6%, how much discount must he pay? What are the proceeds? What are the proceeds for 60 da.?

39. A. D. Redmond has to pay a debt of \$2000 less 10%. He has in the bank \$587.60, and has \$327.50 in cash in his safe. He wishes to leave about \$500 in the bank and about \$100 in his safe. How much, to the nearest \$50, must he borrow? Discount the note for 30 da. at 6%.

Commercial Paper. If a dealer buys some goods for the fall trade, but does not wish to pay for them until after the holidays, he may buy them on credit, giving his note. The manufacturer may need the money at once, in which case he will indorse the note and sell it to a bank or to a note broker for the face less the discount. Such notes are commonly called *commercial paper.*

For example, if you give a manufacturer your note for $500, dated Sept. 1 and due Jan. 1, with interest at 5%, and he, needing the money, discounts the note at a bank Sept. 1 at 6%, what are the proceeds?

Face of the note	$500.
Interest for 4 mo. at 5%	8.33
Amount due at maturity	$508.33
Discount for 4 mo. at 6%	10.17
Proceeds	$498.16

The manufacturer may not need the money Sept. 1, and so he may put the note away in his safe and let it lie there drawing interest. But if he needs the money Sept. 16 he may then decide to discount the note at a bank. We shall then have

Face of the note	$500.
Interest for 4 mo. at 5%	8.33
Amount due at maturity	$508.33
Discount for 107 da. at 6%	9.07
Proceeds	$499.26

Banks usually compute the discount period in days, and the discount by tables based on 360 da. to the year.

If the banks themselves need more money, they may rediscount this paper at the Federal Reserve Bank. The details of the Federal Reserve Bank need not be considered in the schools.

Six Per Cent Method. The following short method, commonly known as the Six Per Cent Method, has been referred to already (page 84), and is convenient not only in computing interest but also in discounting notes.

Find the interest on \$420 for 5 mo. 10 da. at 6%.

Since 2 mo. $= \frac{1}{6}$ yr., the rate for 2 mo. is $\frac{1}{6}$ of 6%, or 1%.

The interest for 2 mo. is 1% of \$420	=	\$4.20
The interest for 2 mo. more	=	4.20
The interest for 1 mo. more is $\frac{1}{2}$ of \$4.20	=	2.10
The interest for 10 da. is $\frac{1}{3}$ of \$2.10	=	.70
The interest for 5 mo. 10 da.	=	\$11.20

Therefore *the interest at 6% for 60 days is 0.01 of the principal, for 6 days is 0.001 of the principal*, and for other periods the interest can be found from this interest.

The rule is conveniently stated as follows:

For 30 da. take $\frac{1}{2}$ of 1% ; for 60 da., 1% ; for 90 da., $1\frac{1}{2}$%.

Since bank notes usually run for 30 da., 60 da., or 90 da., since 6% is the most common rate, and since we can tell the discount for 60 da. by simply glancing at the face of the note, we can often find mentally the discount on bank notes for the usual periods.

Exercise 68. Six Per Cent Method

1. Find the interest on \$4250 for 60 da. at 6%, first by the Six Per Cent Method, then by cancellation, and finally by the ordinary method of finding the interest for 1 yr. and then for the fractional part of a year. Write a statement telling the advantage of the Six Per Cent Method.

2. Using the three methods mentioned in Ex. 1, find the interest at 6% on \$875 for 90 da.; on \$2500 for 30 da.

3. A note for \$1275 is discounted for 60 da. at 6%. Find the discount and the proceeds.

Find the discounts at 6% on notes for the following amounts:

4. \$3000, for 90 da.
5. \$2550, for 90 da.
6. \$4575, for 30 da.
7. \$3575, for 90 da.
8. \$4625, for 90 da.
9. \$8250, for 30 da.
10. \$250, for 3 mo. 8 da.
11. \$800, for 3 mo. 15 da.
12. \$750, for 1 mo. 18 da.
13. \$950, for 3 mo. 20 da.
14. \$2175, for 3 mo. 15 da.
15. \$6500, for 1 mo. 18 da.

16. A note for \$1500 is discounted for 30 da. at 6%. Find the discount and the proceeds.

17. A note for \$3750 is discounted for 90 da. at 6%. Find the discount and the proceeds.

18. A note for \$1250 is discounted for 90 da. at 5%. Find the discount and the proceeds.

Find the discount at 6% and deduct $\frac{1}{6}$ of this.

19. A man wishes to borrow about \$7500 for 60 da. The bank offers to lend it to him at 5%. If he makes out a note for \$7600 and discounts this, how much more than \$7500 will he receive from the bank?

20. A speculator buys some property for \$30,000. He pays \$9600 down and borrows the balance for 90 da. at 5%. How much discount must he pay on the note?

21. How much greater, if any, is the discount on a note for \$2500 discounted for 60 da. at 6% than on one for \$5000 discounted for 30 da. at 6%?

22. A man needs \$9750 to pay for some goods. If he gives a note for \$9800 for 30 da. at 6%, will the proceeds be more or will they be less than the amount he needs? How much more or how much less?

23. A firm gives its note for \$12,500, discounting it for 90 da. at $5\frac{1}{2}$%. How much is the discount?

Exercise 69. Miscellaneous Problems

1. Make out a 60-day note for \$950, dated to-day, payable to M. W. Gross or order at some bank in your vicinity, sign it X. Y. Z., and discount it at 6%.

2. Fill out the blanks in a table like the following and compute the discount at 6% on all the notes mentioned:

Time in Days	To be found	Face of Note				
		\$90	\$280	\$575	\$850	\$1200
30	Discount					
	Proceeds					
60	Discount					
	Proceeds					
90	Discount					
	Proceeds					

3. George Lang sold his farm of 120 A. to Fred Ray at \$95 an acre. Ray paid \$8000 in cash and gave a 90-day note without interest for the balance. If Lang discounted the note at 6% the day it was made, how much did Lang actually receive for the farm?

4. A man deposits \$420 in a savings bank on July 1, \$48.50 on July 19, \$41.30 on Aug. 9, and \$72.90 on Dec. 7. His withdrawals are \$20.50 on July 29, and \$51 on Dec. 22. The next year he deposits \$39.80 on Feb. 4 and \$126.40 on Apr. 14, withdrawing \$38.50 on Feb. 23. The savings bank pays 1% every three months, on Jan. 1, Apr. 1, July 1, and Oct. 1, on the smallest balance in even dollars during the preceding quarter. Find the man's balance on Oct. 1 following his last deposit.

Exercise 70. Review Drill

Write in common numerals:

1. Seven thirty-seconds. 2. Ninety-six thousandths.

Add, and also subtract:

3.	4.	5.	6.
4346.8	9185.48	4878.46	4008.06
3946.8	7369.72	2398.59	869.58

Multiply, and also divide:

7. 259.2 by 2.88. 8. 946.96 by 6.23. 9. 95.19 by 5.01.

Find the sum, difference, product, and both quotients of:

10. $\frac{3}{4}$, $\frac{2}{3}$. 11. $\frac{4}{5}$, $\frac{2}{3}$. 12. $\frac{5}{8}$, $\frac{3}{5}$. 13. $\frac{7}{16}$, $\frac{1}{3}$. 14. $\frac{7}{8}$, $\frac{15}{32}$.

By both quotients of $\frac{3}{4}$ and $\frac{2}{3}$ is meant $\frac{3}{4} \div \frac{2}{3}$ and $\frac{2}{3} \div \frac{3}{4}$.

Find the interest on the following:

15. \$275, 2 yr., 6%. 16. \$275, 2 yr. 8 mo., 6%.

Find the discounts on the following bills:

17. \$475, 6%. 18. \$8734.75, 6%, 3%.

19. Find the discount on \$725 for 60 da. at 4%.

20. Find the interest on \$12,500 for 5 mo. at $4\frac{1}{2}$%.

21. Find the interest on \$675 for 1 yr. 8 da. at 6%.

22. A workman has \$1250 in the savings bank Jan. 1, on which he receives $3\frac{1}{2}$% interest. At the end of 6 mo. he takes out this money and puts the \$1250 with accumulated interest in another bank where he receives 4% interest. How much has he to his credit after the money has been in the second bank for 6 mo.?

Exercise 71. Problems without Numbers

1. Given the face, rate, and time, how do you ascertain the interest due on a note?

2. Which pays the better interest, if the money is left undisturbed for a given number of years, a promissory note or a savings-bank deposit at the same rate per year? Why?

3. How do you fill out a deposit slip? After entering the items, what operation do you perform? How do you make sure that the result is correct?

4. If you know a man's balance in a bank a week ago and his deposits and checks since, how do you find his balance now?

5. How do you find the discount on a promissory note? How do you find the proceeds?

6. If a note drawing a certain rate of interest is discounted on the day it is made, at the same rate, are the proceeds greater than the face, or equal to it, or less? Why is this?

7. How can a manufacturer discount a claim against a purchaser, the claim not being yet due? How is the discount found?

8. If you know the proceeds and the discount, how do you find the face of a note?

9. If you know the face of a note and the proceeds, how do you find the discount?

10. If you know the face of a note, the proceeds, and the time, how do you find the rate of discount?

11. If you have money in a postal savings bank, how much higher rate of interest will you receive if you exchange it for government bonds?

MATERIAL FOR DAILY DRILL

EXERCISE 1

Taking (a) *8.32*, (b) *124.8*, (c) *16.64*, (d) *0.208*, (e) *2.496*, *or* (f) *0.2912*, *as the teacher directs:*

1. Add it to $0.732 + 9 + 7.29 + 68.4 + 1.726 + 0.85$.
2. Subtract it from $25.865 + 21.854 + 78.146$.
3. Multiply it by 125, using a short method.
4. Divide it by 0.13.
5. Find $\frac{7}{8}$ of it; $87\frac{1}{2}\%$ of it.

EXERCISE 2

Taking (a) *$8.64*, (b) *$12.96*, (c) *$17.28*, (d) *$21.60*, (e) *$25.92*, *or* (f) *$38.88*, *as the teacher directs:*

1. Add it to $\$15 + \$0.76 + \$2.88 + \$9.36 + \$2.75$.
2. Subtract it from $\$2.63 + \$8.13 + \$20.75 + \$16.87$.
3. Multiply it by $66\frac{2}{3}$, using a short method.
4. Divide it by \$1.08.
5. Find $12\frac{1}{2}\%$ of it; $37\frac{1}{2}\%$ of it; $62\frac{1}{2}\%$ of it.

This Material for Daily Drill is so arranged as to give daily practice in the fundamental operations. By first going through all the exercises with the number denoted by (a), and then with the one denoted by (b), and so on, more than a hundred different exercises will result, or more than one exercise for each school day of the half year, giving enough for a selection.

EXERCISE 3

Taking (a) *$8.96*, (b) *$13.44*, (c) *$17.92*, (d) *$22.40*, (e) *$26.88*, *or* (f) *$31.36*, *as the teacher directs:*

1. Add it to $19 + $287.30 + $2.75 + $48.60 + $42.86.
2. Subtract it from $4.63 + $10.14 + $27.82 + $9.86.
3. Multiply it by 750, using a short method.
4. Divide it by $1.12.
5. Find 25% of it; $2\frac{1}{2}$% of it; 250% of it.

EXERCISE 4

Taking (a) *$9.28*, (b) *$13.92*, (c) *$18.56*, (d) *$23.20*, (e) *$27.84*, *or* (f) *$32.48*, *as the teacher directs:*

1. Add it to $37.62 + $0.27 + $150 + $3.98 + $48.60.
2. Subtract it from $25.37 + $17.26 + $14.96 + $5.04.
3. Multiply it by 125, using a short method.
4. Divide it by $1.16.
5. Find $\frac{3}{4}$ of it; 75% of it; $\frac{3}{8}$ of it; $37\frac{1}{2}$% of it; 3.75% of it.

EXERCISE 5

Taking (a) *$9.92*, (b) *$14.88*, (c) *$19.84*, (d) *$24.80*, (e) *$29.76*, *or* (f) *$34.72*, *as the teacher directs:*

1. Add it to $3.09 + $17 + $0.75 + $27.68 + $9.32.
2. Subtract it from $2.80 + $15.06 + $19.87 + $10.13.
3. Multiply it by $37\frac{1}{2}$, using a short method.
4. Divide it by $1.24.
5. Divide it by $\frac{4}{5}$; by $2\frac{2}{3}$.

EXERCISE 6

Taking (a) *$10.56*, (b) *$15.84*, (c) *$21.12*, (d) *$26.40*, (e) *$31.68*, *or* (f) *$36.96*, *as the teacher directs:*

1. Add it to $0.29 + $28.70 + $15 + $3.28 + $4.96.
2. Subtract it from $140 + $72.36 + $27.64.
3. Multiply it by $33\frac{1}{3}$, using a short method.
4. Divide it by 8; by 33; by $2.64.
5. Divide it by $16\frac{1}{2}$.

EXERCISE 7

Taking (a) *$10.88*, (b) *$16.32*, (c) *$21.76*, (d) *$27.20*, (e) *$32.64*, *or* (f) *$38.08*, *as the teacher directs:*

1. Add it to $7.33 + $26 + $0.48 + $7.88 + $2.94.
2. Subtract it from $75 + $37.42 + $12.58.
3. Multiply it by 6.25.
4. Divide it by 8; by 17; by 34; by $2.72; by $1.36.
5. Divide it by $6\frac{4}{5}$; by $11\frac{1}{3}$.

EXERCISE 8

Taking (a) *$11.20*, (b) *$16.80*, (c) *$22.40*, (d) *$33.60*, (e) *$28*, *or* (f) *$39.20*, *as the teacher directs:*

1. Add it to 9 times itself, using a short method.
2. Subtract it from 11 times itself.
3. Multiply it by $37\frac{1}{2}$.
4. Divide it by 8; by 7; by 5; by 35; by $1.40.
5. Divide it by $8\frac{3}{4}$; by $4\frac{3}{8}$; by $2\frac{3}{16}$.

EXERCISE 9

Taking (a) *$11.52*, (b) *$17.28*, (c) *$23.04*, (d) *$28.80*, (e) *$34.56*, *or* (f) *$40.32*, *as the teacher directs:*

1. Add it to $1.20 + $0.92 + $17 + $3.75 + $28.67.
2. Subtract it from $32.75 + $19.82 + $10.18.
3. Multiply it by 62.5.
4. Divide it by 2; by 4; by 8; by 16; by 32; by 36.
5. Multiply it by $\frac{3}{8}$. Divide it by $2\frac{2}{3}$.

EXERCISE 10

Taking (a) *$11.84*, (b) *$17.76*, (c) *$23.68*, (d) *$29.60*, (e) *$35.52*, *or* (f) *$41.44*, *as the teacher directs:*

1. Add it to $12 + $16.75 + $0.82 + $2.98 + $48.20.
2. Subtract it from $75 + $37.80 + $42.60 + $17.90.
3. Multiply it by 37.85.
4. Divide it by 2; by 4; by 8; by $0.37.
5. Multiply it by $0.12\frac{1}{2}$; by $\frac{1}{8}$; by $12\frac{1}{2}\%$.

EXERCISE 11

Taking (a) *1216*, (b) *182.4*, (c) *24.32*, (d) *0.4256*, (e) *3.648*, *or* (f) *0.304*, *as the teacher directs:*

1. Add it to $9 + 15.75 + 2\frac{1}{2} + 5\frac{3}{4}$.
2. Subtract it from 1300.
3. Multiply it by 0.365.
4. Divide it by 3.04; by 0.7; by 40; by 400; by 4000.
5. Divide it by $12\frac{2}{3}$; by $6\frac{1}{3}$; by $3\frac{1}{6}$.

EXERCISE 12

Taking (a) *124.8*, (b) *18.72*, (c) *24.96*, (d) *0.312*, (e) *3.744*, *or* (f) *0.4368*, *as the teacher directs:*

1. Add it to $3.848 + 148.276 + 175 + 48.76 + 9.009$.
2. Subtract it from $4.6273 + 74.896 + 56.215$.
3. Multiply it by 12.5, using a short method.
4. Divide it by 2; by 4; by 8; by 13; by 3.12; by $6\frac{1}{2}$.
5. Find $12\frac{1}{2}\%$ of it, using a short method.

EXERCISE 13

Taking (a) *13.44*, (b) *20.16*, (c) *2.688*, (d) *0.4704*, (e) *4.032*, *or* (f) *0.336*, *as the teacher directs:*

1. Add it to $72.8796 + 182.08 + 7.087 + 72.6 + 0.983$.
2. Subtract it from $2.786 + 46.93 + 53.17$.
3. Multiply it by 342.87.
4. Divide it by 3; by 7; by 8; by 0.042; by 3.36.
5. Of what number is it 75%? $\frac{3}{4}$? $7\frac{1}{2}\%$? $\frac{3}{4}\%$?

EXERCISE 14

Taking (a) *1.408*, (b) *21.12*, (c) *2.816*, (d) *0.352*, (e) *4.224*, *or* (f) *0.4928*, *as the teacher directs:*

1. Add it to $482.76894 + 9 + 0.987 + 0.7236 + 483$.
2. Subtract it from $0.7 + 276.93 + 14.963 + 5.037$.
3. Multiply it by $\frac{3}{4}$ of $\frac{1}{4}$.
4. Divide it by 0.8; by 4.4; by 0.352; by 2.2; by $\frac{4}{5}$.
5. Of what number is it 80%? 120%? $1\frac{1}{5}$? $\frac{6}{5}$?

EXERCISE 15

Taking (a) *15.36*, (b) *23.04*, (c) *3.072*, (d) *0.384*, (e) *4.608*, *or* (f) *0.5376*, *as the teacher directs:*

1. Add it to $0.2702 + 298.742 + 0.7298 + 7017 + 2983$.
2. Subtract it from $36.7 + 921.006 + 78.239 + 21.761$.
3. Multiply it by $122\frac{1}{4}$.
4. Divide it by $9\frac{3}{5}$.
5. Of what number is it 125%? $1\frac{1}{4}$? $\frac{5}{4}$? $12\frac{1}{2}\%$?

EXERCISE 16

Taking (a) *15.68*, (b) *2.352*, (c) *313.6*, (d) *0.392*, (e) *470.4*, *or* (f) *5488*, *as the teacher directs:*

1. Add it to $0.1271 + 2789.762 + 2936 + 7064 + 0.8729$.
2. Subtract it from $48.789 + 968.32 + 3429 + 6571$.
3. Multiply it by itself.
4. Divide it by 0.0784.
5. What per cent is it of 5 times itself? of half itself?

EXERCISE 17

Taking (a) *2.88*, (b) *26.4*, (c) *124.8*, (d) *17.04*, (e) *34.56*, *or* (f) *69.12*, *as the teacher directs:*

1. Add it to 125% of itself.
2. Subtract it from 200% of itself.
3. Multiply it by 0.5% of itself.
4. Divide it by 0.15; by 0.075; by 1.5.
5. Of what number is it 12%? 1.2%? 120%?

PART II. GEOMETRY

I. GEOMETRY OF FORM

First Steps in Geometry. Thousands of years ago, when people began to study about forms, they were interested in pictures showing the shapes of objects; these they used in decorating their walls, and later in showing the plans of their houses and their temples and in representing animals and human beings. As land became valuable they showed an interest in measuring objects, fields, and building material. When they wished to locate places on the earth's surface and when they began to study the stars, it was necessary that they should consider position. From very early times, therefore, the ideas of form, size, and position have interested humanity.

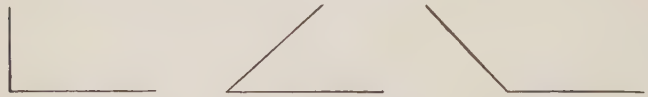
There are three things which we naturally ask about an object: What is its shape? How large is it? Where is it? It is these three questions that form the bases of the kind of geometry which we are now about to study. There are also other questions which we might ask about the object, such as these: How much is it worth? What is its color? Of what is it made? None of these questions, however, has to do with geometry.

The teacher will recognize that demonstrative geometry is not touched upon directly by the three questions above set forth. Another question might be asked relating to all three, namely, How do you know that your statement is true? It is this question which leads to the proof of propositions. For the present we are concerned almost exclusively with intuitional and observational geometry as related to the questions of shape, size, and position.

Geometric Figures. You are already familiar with such common forms as the square, triangle, circle, arc, and cube. Such forms are generally known as *geometric figures.*

Angle. Two straight lines drawn from a point form an *angle.* The two straight lines are called the *sides* of the angle, and the point where they meet is called the *vertex.*

The three most important angles are the *right angle*, the *acute angle*, which is less than a right angle, and the *obtuse angle*, which is greater than a right angle.

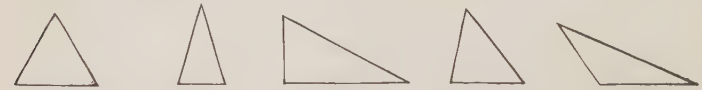


RIGHT ANGLE ACUTE ANGLE OBTUSE ANGLE

If necessary, the teacher should explain what we mean when we say that an angle is greater than or less than another angle. This is easily done by slowly opening a pair of compasses.

Acute angles and obtuse angles are called *oblique angles.*

Triangle. A figure bounded by three straight lines is called a *triangle.*



EQUILATERAL ISOSCELES RIGHT ACUTE OBTUSE

The five most important kinds of triangles are the *equilateral triangle*, having all three sides equal; the *isosceles triangle*, having two sides equal; the *right triangle*, having one right angle; the *acute triangle*, having three acute angles; and the *obtuse triangle*, having one obtuse angle.

The side opposite the right angle in a right triangle is called the *hypotenuse* of the right triangle.

The sum of the sides of a triangle is called the *perimeter.*

Exercise 1. Angles and Triangles

Examples 1 to 6, oral

1. Point to three right angles in the room.

2. Point, if possible, to two straight lines on the wall or on a desk which form an acute angle.

3. Point, if possible, to two straight lines in the schoolroom which form an obtuse angle.

4. Which is the greater, an acute angle or an obtuse angle?

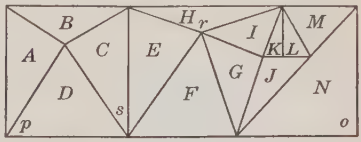
5. How many right angles all lying flat on the top of a table will completely fill the space around a point on the table?

6. If one side of an equilateral triangle is 6 in., what is the perimeter of the triangle?

7. Draw a right angle as accurately as you can by the aid of a ruler.

8. Draw an acute angle and an obtuse angle, writing the name under each.

9. In this figure name by capital letters the triangles which seem to you to be right triangles.



10. In the same figure name by a small letter each of the acute angles.

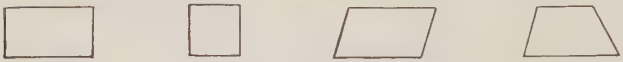
11. In the same figure name by a capital letter each of the obtuse triangles.

12. What kinds of angles are represented in the figure by the letters *o*, *p*, *r*, *s*? Write the name after each letter.

13. If two straight lines intersect, what can you say as to any equal angles?

Quadrilateral. A figure bounded by four straight lines is called a *quadrilateral.*

The rectangle, square, parallelogram, and trapezoid, the four most important kinds of quadrilaterals, are shown below.



RECTANGLE SQUARE PARALLELOGRAM TRAPEZOID

A quadrilateral which has all its angles right angles is called a *rectangle.*

A rectangle which has its sides all equal is called a *square.*

A quadrilateral which has its opposite sides parallel is called a *parallelogram.*

A quadrilateral which has one pair of opposite sides parallel is called a *trapezoid.*

It is not necessary at this time to give a formal definition of parallel lines. The students are familiar with the term.

We shall hereafter use the word *line* to mean straight line unless we wish to use the word *straight* with *line* for purposes of emphasis.

Polygon. A figure bounded by straight lines is called a *polygon.* The quadrilaterals shown above are all special kinds of polygons, and a triangle is also a polygon.

Polygons may have three, four, five, six, or any other number of sides greater than two.

The side on which a polygon appears to rest is called the *base* of the polygon.

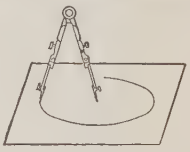
The sum of all the sides of a polygon is called the *perimeter* of the polygon.

The points in which each pair of adjacent sides intersect are called the *vertices* of the polygon.

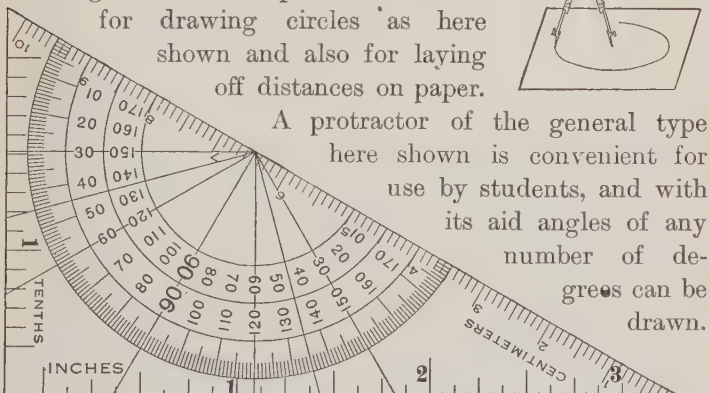
In the case of a triangle the vertex of the angle opposite the base is usually called the *vertex* of the triangle.

Congruent Figures. If two figures have exactly the same shape and size, they are called *congruent figures.*

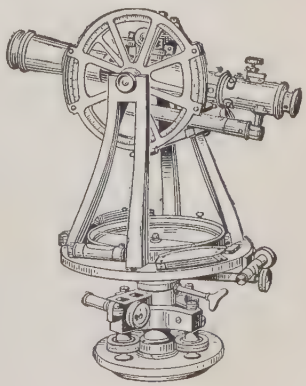
Drawing Instruments. The instruments commonly used in drawing the figures in geometry are the compasses, the ruler, the protractor, and the right triangle. The compasses are used for drawing circles as here shown and also for laying off distances on paper.



A protractor of the general type here shown is convenient for use by students, and with its aid angles of any number of degrees can be drawn.



For work out of doors a surveyor measures angles and finds levels by means of a transit such as is here shown.



Each student should have a ruler, a pair of compasses, and a protractor, since the constructions studied in this book can be made only by their use.

If necessary such familiar terms as *circle, radius, diameter, arc,* and *circumference* should be explained informally. They are more formally stated later.

On pages 117 and 119 and later in the work some interesting illustrations of ancient instruments are given. Students often make similar instruments for use in geometry.

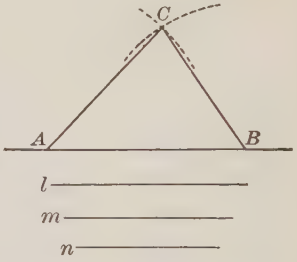
Constructing Triangles. We often have to construct triangles of various shapes and sizes. We shall first consider the following case:

Construct a triangle having its sides respectively equal to three given lines.

Let l, m, n be the given lines.

It is required to construct a triangle with l, m, n as sides.

Draw a line with the ruler and on it mark off with the compasses a line AB equal to l.



It is more nearly accurate to do this with the compasses than with a ruler.

With A as center and m as radius draw a circle; with B as center and n as radius draw another circle cutting the first at C. Draw AC and BC.

Then because $AB = l$, $AC = m$, and $BC = n$ it follows that ABC is the required triangle.

Show why it is not necessary to draw the whole circle in either case.

Teachers should informally explain to the students the methods commonly used in lettering a line, an angle, and a triangle.

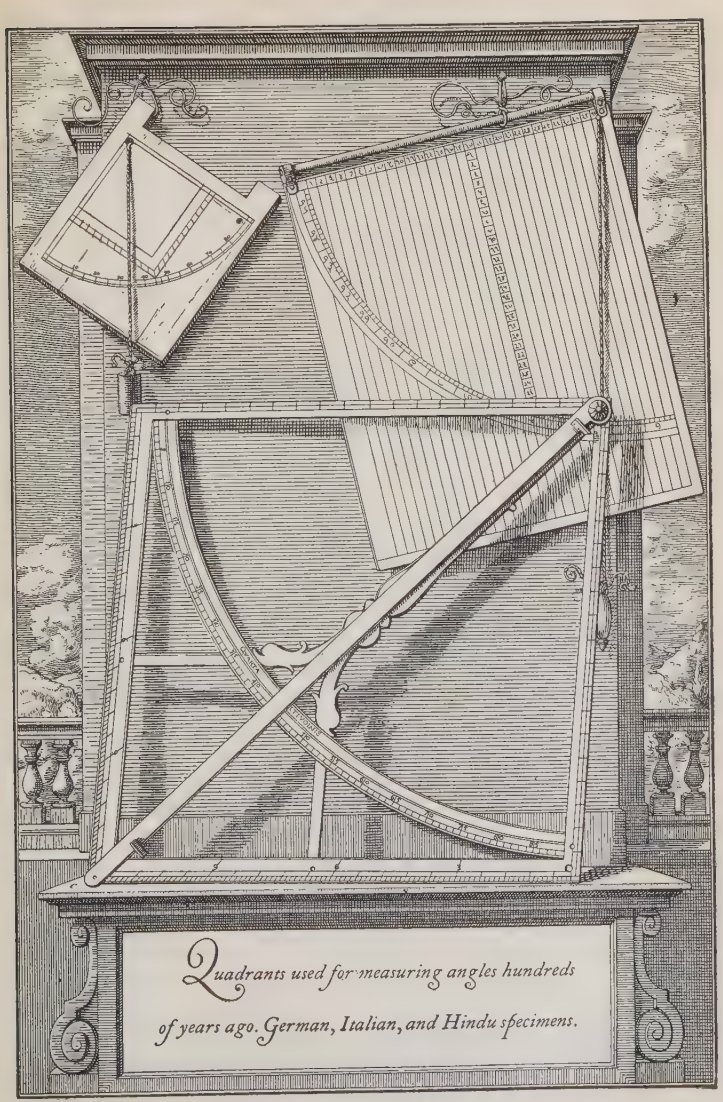
Exercise 2. Triangles

1. Construct a triangle with sides 2 in., 3 in., 4 in.

Construct triangles with sides as follows:

2. 3 in., 4 in., 5 in.
3. $1\frac{1}{2}$ in., 2 in., $2\frac{1}{2}$ in.
4. $1\frac{1}{4}$ in., $2\frac{1}{4}$ in., 3 in.
5. $\frac{1}{2}$ in., $\frac{3}{4}$ in., 1 in.
6. $2\frac{1}{2}$ in., $2\frac{1}{2}$ in., $2\frac{1}{2}$ in.
7. 3 in., $3\frac{1}{4}$ in., $3\frac{1}{4}$ in.

In schools in which the metric system is taught it is desirable to use the system in this work. The necessary metric measures often will be found on protractors such as the one shown on page 115.



Quadrants used for measuring angles hundreds of years ago. German, Italian, and Hindu specimens.

Isosceles Triangle. In the case studied on page 116 we see that the three sides need not all be equal. If two sides are equal we have to construct an isosceles triangle.

Construct a triangle having two sides each equal to a given line and the base equal to another given line.

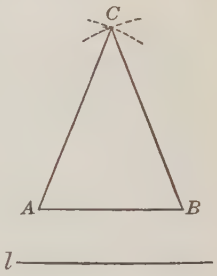
The *base* of an isosceles triangle is always taken as the side which is not equal to one of the other sides.

Let AB be the given base and let l be the given line.

Then with center A and radius l draw a circle, and with center B and radius l draw another circle, or preferably only an arc in each case.

Let the two arcs or the two circles intersect at the point C.

Then ABC is the triangle required.



Equilateral Triangle. From the preceding case we see that if the base is equal to each of the other sides, we shall have an equilateral triangle.

Exercise 3. Isosceles and Equilateral Triangles

1. In making a pattern for the tiles used in the floor shown below it is necessary to draw an equilateral triangle of side 1 in. Draw such a triangle.

Construct isosceles triangles with bases 1 in. and equal sides as follows:

2. $\frac{3}{4}$ in. 3. $\frac{7}{8}$ in. 4. $1\frac{1}{2}$ in. 5. 2 in. 6. $2\frac{1}{8}$ in.

Construct equilateral triangles with sides as follows:

7. $\frac{3}{4}$ in. 8. $\frac{7}{8}$ in. 9. $1\frac{1}{8}$ in. 10. $1\frac{7}{8}$ in. 11. $2\frac{1}{4}$ in.

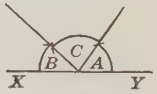
A curious illustration
from an Italian work of the seventeenth century
showing the use of the ancient quadrant.
The distance was required
for the purpose of properly fixing the guns.
The computations may be made
in various ways.

12. Cut three isosceles triangles of different shapes from paper and fold each through the middle so that one of the equal sides lies exactly on the other. What inference can you make as to the equality or inequality of the angles which are opposite the equal sides? Write the statement as follows: In an isosceles triangle the angles opposite the equal sides are equal.

13. Draw three equilateral triangles of different sizes. With a protractor measure each angle in each of the triangles. What inference can you make as to the number of degrees in each angle? Write the statement, beginning as follows: The number of degrees in each angle of, etc.

14. From Ex. 13 what inference can you make as to the number of degrees in the sum of the three angles of an equilateral triangle? This is the same as the number of degrees in how many right angles?

15. Draw three triangles of various shapes and investigate for each the conclusion drawn in Ex. 14. This is most easily done by cutting them from paper and then cutting off the three angles in each case and fitting them together. Write the statement, beginning as follows: In any triangle the sum of the three angles is equal to, etc.



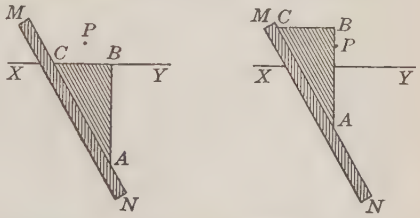
16. From the truth discovered in Ex. 15, find the third angle of a triangle in which two angles are 75° and 45°.

17. In a certain right triangle one acute angle is 30°. How many degrees are there in the other acute angle?

In this work the student is led to discover by experiment various important propositions to be proved later in his work in geometry. Teachers may occasionally find it advantageous to develop simple proofs in connection with this intuitional treatment.

Perpendicular. A line which makes a right angle with another line is said to be *perpendicular* to that line.

One of the best practical methods of constructing a line perpendicular to a given line and passing through a given point is shown in this illustration.



Place a right triangle ABC so that BC lies along the given line. Lay a straightedge or ruler along AC, as in the left-hand figure. Since you wish the perpendicular line, or *perpendicular*, to pass through the point P, slide the triangle along MN until AB passes through the point P, as shown in the right-hand figure. Then draw a line along AB, and it will be perpendicular to the line XY and will pass through the point P.

Exercise 4. Perpendiculars

1. Draw a line XY and mark a point P about $\frac{1}{2}$ in. below it. Through P construct a line perpendicular to XY, by the above method.

2. Through a point P on the line XY construct a line perpendicular to XY, by the above method.

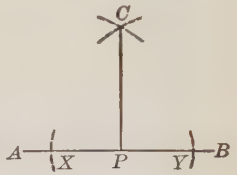
3. Construct a right triangle in which the two shorter sides shall be $1\frac{1}{2}$ in. and 2 in.

4. Construct a square having its side 2 in.

5. Draw a picture showing how two carpenter's squares can be tested by standing them on any flat surface with two edges coinciding and two other edges extending in opposite directions.

Other Methods of Constructing Perpendiculars. There are other convenient methods of constructing perpendiculars.

From a given point on a given straight line construct a perpendicular to the line.

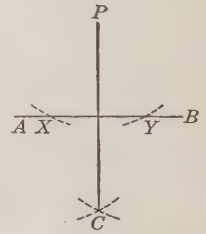


Let AB be the given line and P be the given point.

With P as center and with any convenient radius draw arcs intersecting AB at X and Y.

With X as center and XY as radius draw a circle, and with Y as center and the same radius draw another circle, and call one intersection of the circles C.

With a ruler draw a line from P to C.



From a given point outside a given straight line construct a perpendicular to the line.

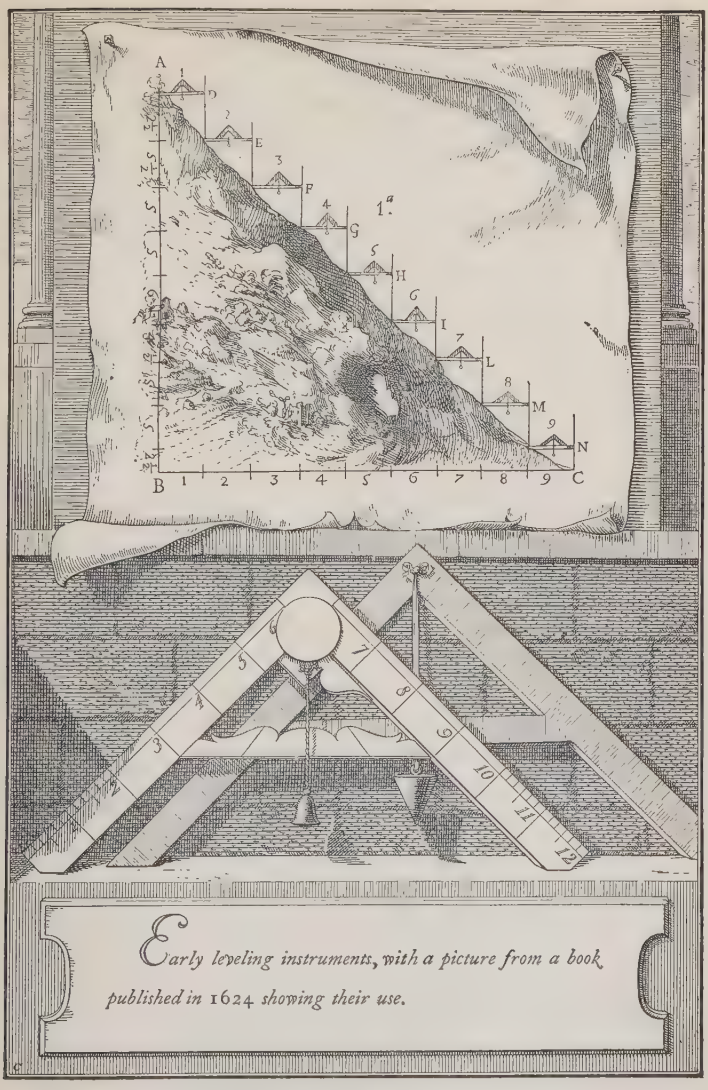
Let AB be the given line and P be the given point. How are the points X and Y fixed? Then how is the point C fixed? Draw the perpendicular PC.

Exercise 5. Perpendiculars

1. In making a pattern for a tiled floor like the one here shown it becomes necessary to draw a square 1 in. on a side. Construct such a square, using the first of the above methods.

2. Construct a rectangle as in Ex. 1, using the second of the above methods.

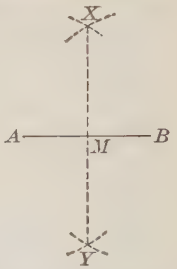
3. Given two points on a given line, construct perpendiculars to the line from each of them.



Early leveling instruments, with a picture from a book published in 1624 *showing their use.*

JM1

Bisecting a Line. To divide a line into two equal parts is to *bisect* it. In constructing the common figures we often have to bisect a line. We can bisect a line roughly by measuring it with a ruler, but for accurate work we have a much better method.



Bisect a given line.

Let AB be the given line. What is now required? With A and B as centers and with radius greater than $\frac{1}{2}AB$ draw arcs. The most convenient radius is usually AB itself.

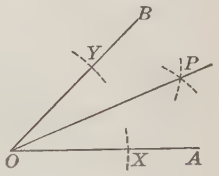
Call the points of intersection X and Y. Draw the straight line XY, and call the point where it cuts the given line M.

Then XY bisects AB at M.

This is much more nearly accurate than it is to measure the line with a ruler and then take half the length.

Bisecting an Angle. To draw a line from the vertex of an angle dividing it into two equal angles is to *bisect* it.

Bisect a given angle.



Let AOB be the given angle.

What is now required?

With O as center and with any convenient radius draw an arc cutting OA at X and OB at Y.

With X and Y respectively as centers and with a radius greater than half the distance from X to Y draw arcs and call their point of intersection P. Draw OP.

Then OP is the required bisector.

This is much more nearly accurate than it is to measure the angle with a protractor and then take half the number of degrees.

Exercise 6. Simple Constructions

1. Draw a line 4.5 in. long. Bisect this line with ruler and compasses. Check the construction by folding the paper at the point of bisection, making a fine pinhole through one end of the line to see if it strikes the other end.

2. Construct a triangle having two of its sides 3 in., the third side being less than 6 in.

3. Construct a triangle having its sides respectively 2 in., 2.5 in., and 3 in.

4. Draw a line 4 in. long, and at a point 1 in. from either end construct a perpendicular to the line.

5. Is it possible to construct a triangle having its sides respectively 3 in., 2 in., and 1 in.? If not, what is there in the general nature of these lengths which makes such a triangle impossible?

6. With a protractor draw an angle of 35°. Bisect this angle and check the work with the protractor.

To draw the angle of 35° draw a line, mark a point O upon it, lay the hypotenuse of a triangular protractor on it, sliding it down slightly so that the center of the circle rests on O. Lay a ruler on the protractor from O along the line of 35° and mark a point on the paper. Remove the protractor and draw a line from O to the point.

Construct the triangles whose sides are as follows and bisect all three angles of each triangle:

7. 4 in., 3 in., $4\frac{1}{2}$ in.

8. 5 in., 7 in., 8 in.

9. 6 in., 3 in., 5 in.

10. $3\frac{1}{2}$ in., $4\frac{1}{2}$ in., $5\frac{1}{2}$ in.

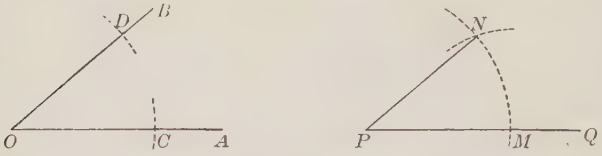
11. $7\frac{1}{2}$ in., $4\frac{1}{2}$ in., 5 in.

12. $3\frac{1}{2}$ in., $3\frac{1}{2}$ in., $3\frac{1}{2}$ in.

13. In Exs. 7–12 what do you observe as to the way in which the three bisectors meet? Write a statement of your conclusion, beginning as follows: The bisectors of the three angles of a triangle, etc.

Constructing an Angle equal to a Given Angle. In copying figures we often have to construct an angle equal to a given angle. This leads to the following construction:

From a given point on a given line construct a line which shall make with the given line an angle equal to a given angle.



Let P be the given point on the given line PQ and let angle AOB be the given angle.

What is now required?

With O as center and with any radius draw an arc cutting OA at C and OB at D.

With P as center and with OC as radius draw an arc cutting PQ at M.

With M as center and with the straight line joining C and D as radius draw an arc cutting the arc just drawn at N, and draw PN.

Then the angle MPN is the required angle.

Exercise 7. Simple Constructions

Construct triangles with sides as follows and bisect all three of the sides of each triangle:

1. 5 in., 6 in., 7 in.
2. 4 in., 4 in., 7 in.
3. $3\frac{1}{2}$ in., $4\frac{1}{2}$ in., $7\frac{1}{2}$ in.
4. 3 in., $3\frac{1}{2}$ in., $4\frac{1}{2}$ in.
5. $2\frac{1}{4}$ in., 4 in., 4 in.
6. $3\frac{1}{2}$ in., $3\frac{1}{2}$ in., 4 in.

Interesting figures may be formed by connecting the points of bisection and shading in various ways the parts thus formed.

7. Construct a triangle ABC with $AB = 1$ in., $AC = 1\frac{1}{4}$ in., angle $A = 30°$, and then construct another triangle XYZ with $XY = 1$ in., $XZ = 1\frac{1}{4}$ in., angle $X = 30°$. Are the triangles ABC and XYZ congruent?

8. From Ex. 7 write a complete statement of the truth inferred, beginning as follows: Two triangles are congruent if two sides and the included angle of one are respectively equal to, etc.

9. Construct a triangle ABC in which angle $A = 30°$, angle $B = 60°$, $AB = 1\frac{1}{2}$ in., and then construct another triangle XYZ in which angle $X = 30°$, angle $Y = 60°$, $XY = 1\frac{1}{2}$ in. Are these triangles congruent? What is the reason? Write a complete statement of the truth inferred, as in Ex. 8.

10. Construct a triangle ABC in which $AB = 1$ in., $BC = 1\frac{1}{4}$ in., $CA = 1\frac{1}{2}$ in., and then construct another triangle XYZ in which $XY = 1$ in., $YZ = 1\frac{1}{4}$ in., $ZX = 1\frac{1}{2}$ in. Are these triangles congruent? Write a complete statement of the truth inferred, as in Ex. 8.

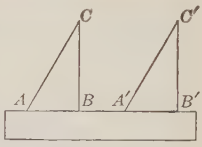
11. Construct a triangle with angles 30°, 60°, and 90°, and another triangle with sides twice as long but with angles the same. Are these triangles congruent? Are triangles in general congruent if the angles of one are respectively equal to the angles of the other?

12. As in Ex. 11, construct two triangles with angles 45°, 45°, and 90°, one with sides three times as long as the other.

13. Try to construct a triangle with angles 45°, 60°, and 90°. If you have any difficulty in making the construction, write a statement of the cause.

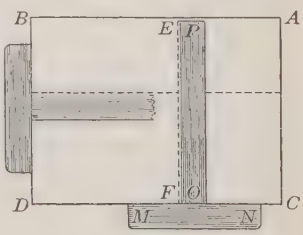
14. Try to construct a triangle with angles 45°, 45°, and 100°. If you have any difficulty in making the construction, write a statement of the cause.

Parallel Lines. One of the most common constructions in making architectural and mechanical drawings is to draw one line parallel to another line. For practical purposes one of the best plans is to place a wooden or celluloid triangle ABC with one side BC on the given line, lay a ruler along another side AB, and then slide the triangle along the ruler to the position $A'B'C'$ (read A-prime, B-prime, C-prime). Then $B'C'$ is parallel to BC.



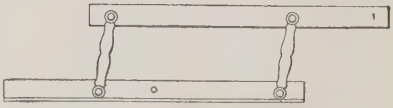
A triangular protractor like the one shown on page 115 of this book may be used for the above purpose.

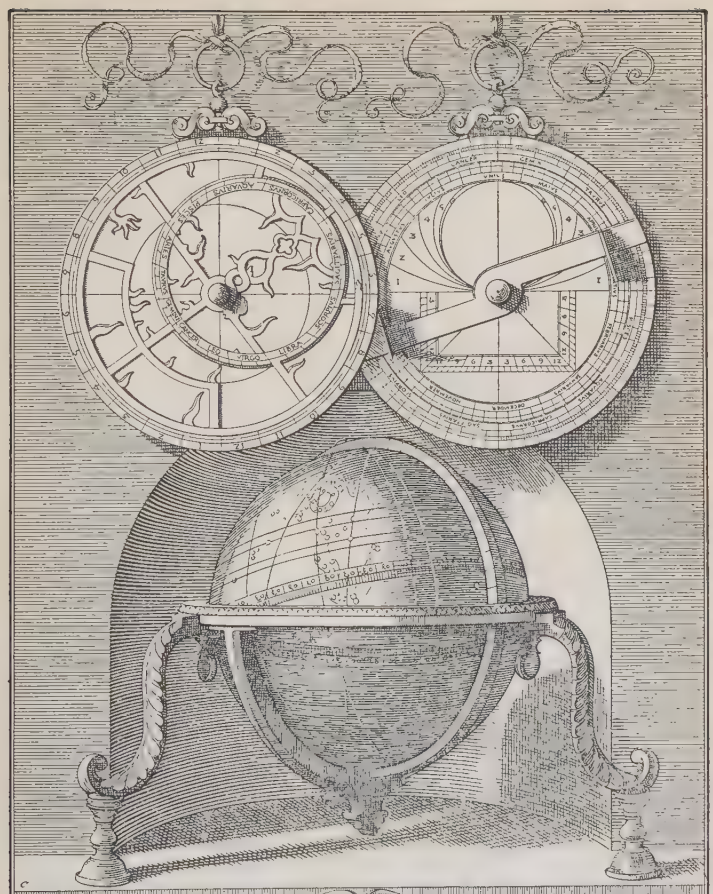
Draftsmen in offices of architects or in machine shops often use a T-square as here shown. As the part MN slides along the edge CD of a drawing board, the part OP moves parallel to its original position. Drawing EF and sliding the T-square along, we can easily draw lines parallel to EF. A second T-square may slide along BD if the board is rectangular, and thus lines can be drawn perpendicular to the line EF or to any lines parallel to it.



When the lines are very long, this is the best method.

Draftsmen also use a parallel ruler like the one here shown. They also use a cylindric ruler, rolling it along the paper as a guide for parallel lines. In general, however, the plan of sliding a triangle along a ruler is one of the simplest and at the same time is accurate. It should be used in the exercises which follow.

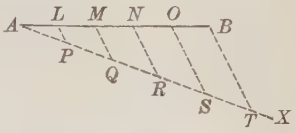




Early uses of geometry in studying the stars.
Above, an astrolabe used in measuring the angles of stars above the horizon. Below, an ancient Hindu bronze sphere of the heavens, with stars inlaid in silver.

Dividing a Line. We often need to divide a line into a given number of equal parts; that is, to solve this problem:

Divide a given line into any given number of equal parts.



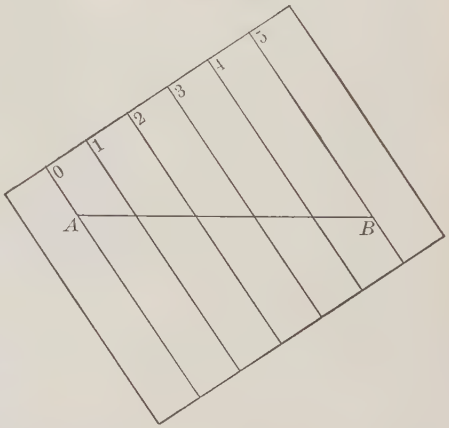
Let AB be the given line, and let it be required to divide AB into five equal parts.

Draw any line from A, as AX. Mark off on AX with the compasses any five equal lengths AP, PQ, QR, RS, and ST.

Draw TB, and then, by sliding a triangle along a ruler, draw SO, RN, QM, and PL parallel to TB.

Then AB is divided into five equal parts, AL, LM, MN, NO, and OB.

The material for another very simple method may be easily prepared by the student. Let him rule a large sheet of paper with several parallel lines at equal intervals, and number these lines as shown on the edge. If it is desired to divide the line AB into five equal parts, place the paper on which AB is drawn over the ruled paper so that the line 0 passes through A and the line 5 through B. Lay the ruler along each ruled line in turn and mark each point of division. In this way the four required points of division may be accurately found.



Exercise 8. Simple Constructions

1. Draw a line 5 in. long and divide it into nine equal parts by using ruler, triangle, and compasses.

Construct triangles whose sides are as follows, and construct a perpendicular to each side at its midpoint:

2. $4\frac{1}{2}$ in., $4\frac{3}{4}$ in., 5 in.

3. $3\frac{1}{2}$ in., $3\frac{1}{4}$ in., $5\frac{1}{4}$ in.

4. $5\frac{1}{2}$ in., $5\frac{3}{4}$ in., $6\frac{1}{4}$ in.

5. 4 in., $5\frac{1}{2}$ in., $6\frac{1}{4}$ in.

The teacher should ask for the inference as to the meeting of the three perpendicular bisectors of the sides of a triangle.

6. With a protractor draw an angle of 45°. With ruler and compasses bisect this angle. Check the construction by folding the paper; by using the protractor.

7. Draw any triangle, bisect the sides, and join the points of bisection, thus forming another triangle. With ruler and triangle test to see whether the sides of the small triangle are parallel to those of the large triangle.

8. Repeat Ex. 7 for a triangle of different shape. What general law do you infer from these two cases?

9. Draw a line $4\frac{1}{8}$ in. long and divide it into seven equal parts.

10. Construct a square 3 in. on a side. If the figure is correctly drawn, the two diagonals will be equal. Check by measuring the diagonals with the compasses.

If such words as *diagonal* are not familiar they should be explained by the teacher when they are met. It is desirable to avoid formal definitions at this time, provided the students use the terms properly.

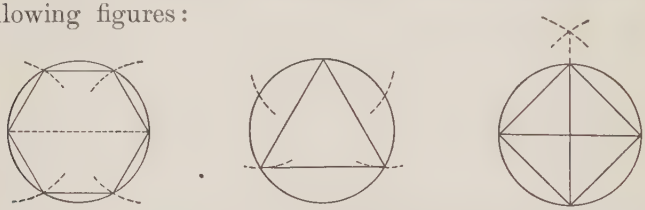
11. Construct two parallel lines and draw a slanting line cutting these lines so that eight oblique angles are formed. Name the various pairs of angles in the figure that appear to be equal.

Geometric Patterns. By the aid of the constructions described on pages 116–130 it is possible to construct a large number of useful and interesting patterns, designs for decorations, and plans for buildings or gardens.

To secure the best results in this work the pencil should be sharpened to a fine point and should contain rather hard lead, and the lines should be drawn very fine.

Exercise 9. Geometric Patterns

1. By the use of compasses and ruler construct the following figures:

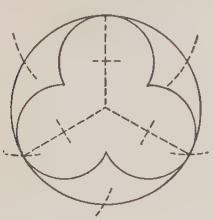


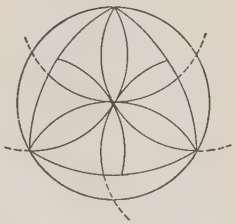
The lines made of short dashes show how to fix the points needed in drawing a figure, and they should be erased after the figure is completed unless the teacher directs that they be retained to show how the construction was made.

2. By the use of compasses and ruler construct the following figures:

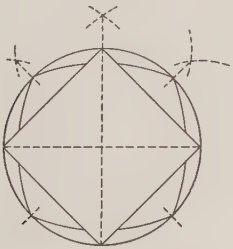
It is apparent from the figures in Exs. 1 and 2 that the radius of the circle may be used in drawing arcs which shall divide the circle into six equal parts by simply stepping round it.

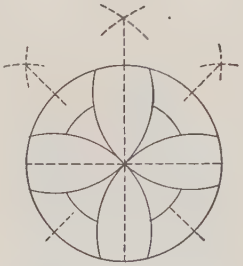
3. By the use of compasses and ruler construct the following figures, shading such parts as will make a pleasing design in each case:



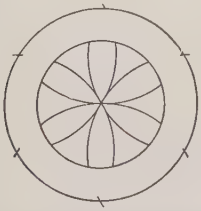


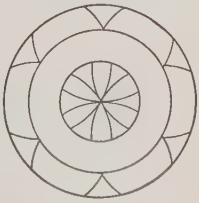
4. By the use of compasses and ruler construct the following figures, shading such parts as will make a pleasing design in each case:





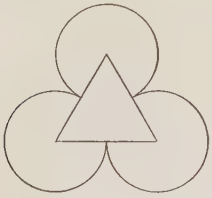
5. By the use of compasses and ruler construct the following figures:





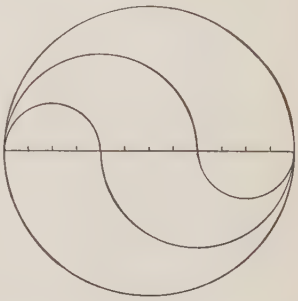
In such figures artistic patterns may be made by coloring portions of the drawings. In this way designs are made for stained-glass windows, for oilcloths, for colored tiles, and for other decorations.

6. By the use of compasses and ruler construct the following figures, leaving the dotted construction lines:



As stated on page 133, artistic patterns may be made by coloring various parts of these drawings. Interesting effects are also produced in black and white, as in the designs in Ex. 9 on page 135.

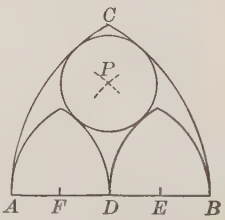
7. Draw a line $1\frac{1}{2}$ in. long and divide it into eighths of an inch, using the ruler. Then with the compasses construct this figure.



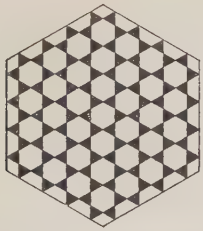
It is easily shown, when we come to the measurement of the circle, that these two curve lines divide the space inclosed by the circle into parts that are exactly equal.

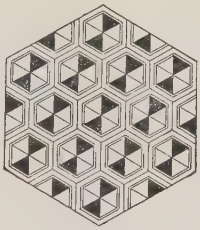
By continuing each semicircle to make a complete circle another interesting figure is formed. Other similar designs are easily invented, and students should be encouraged to make such original designs.

8. In planning a Gothic window this drawing is needed. The arc BC is drawn with A as center and AB as radius. The small arches are drawn with A, D, and B as centers and AD as radius. The center P is found by using A and B as centers and AE as radius. How may the points D, E, and F be found? Draw the figure.



9. Copy each of the following designs, enlarging each to twice the size shown on this page:





This example and the following examples on this page may be omitted by the class at the discretion of the teacher if there is not enough time for such work in geometric drawing.

10. This figure shows a piece of inlaid work in an Italian church. Construct a design of this general nature, changing it to suit your taste. Construct the figures as accurately as you can.

11. Construct a design for parquetry flooring, using only combinations of squares.

12. Repeat Ex. 11, using combinations of squares and equilateral triangles.

13. Repeat Ex. 11, using combinations of squares, rectangles, and equilateral triangles.

14. Construct a design for a geometric pattern for linoleum, using only combinations of circles and squares.

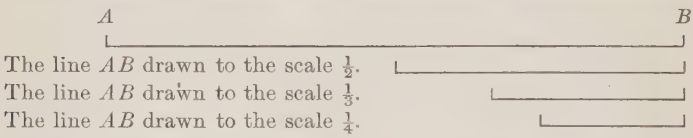
15. Repeat Ex. 14, using only combinations of circles and equilateral triangles.

16. Repeat Ex. 14, using only combinations of circles, squares, and equilateral triangles.

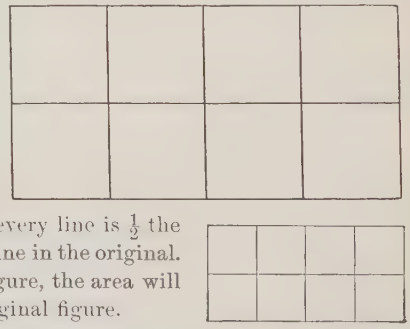
Drawing to Scale. The ability to understand drawings, maps, and other graphic representations depends in part upon knowing how to draw to scale.

Thus, if your schoolroom is 30 ft. long and 20 ft. wide, and you make a floor plan 3 in. long and 2 in. wide, you draw the plan to scale, 1 in. representing 10 ft. We indicate this by writing: "Scale, 1 in. = 10 ft." We may also write this: "Scale, 1 in. = 120 in.," or "Scale $\frac{1}{120}$." We often write $1'$ for 1 ft. and $1''$ for 1 in., so that the scale may also be indicated as $1'' = 10'$.

The following shows a line AB drawn to different scales:



The figures shown below illustrate the drawing of a rectangle to scale. In this case the lower rectangle is a drawing of the upper one to the scale $\frac{1}{2}$, or 1 to 2, or $1''$ to $2''$.



Notice that the area of the lower rectangle is only $\frac{1}{4}$ that of the upper one. When we draw to the scale $\frac{1}{2}$ we mean that the length of every line is $\frac{1}{2}$ the length of the corresponding line in the original. Whatever the shape of the figure, the area will then be $\frac{1}{4}$ the area of the original figure.

Maps are figures drawn to scale. The scale is usually stated on the map, as you will see in any geography. The scale used on a map is often expressed by means of a line divided to represent miles, and sometimes by such a statement as that 1 in. = 100 mi.

Exercise 10. Drawing to Scale

1. Measure the cover of this book. Draw the outline to the scale $\frac{1}{6}$.

This means that the four edges are to be drawn to form a rectangle like the front cover, with no decorations.

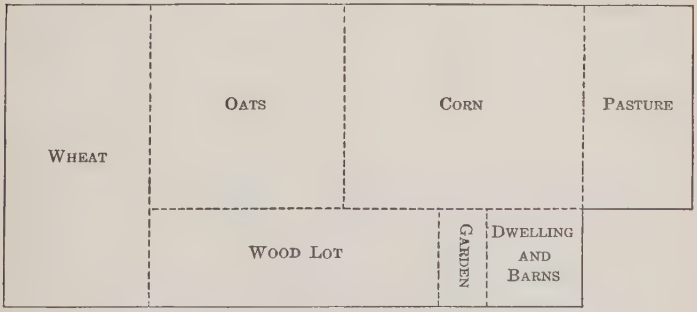
2. Measure the top of your desk. Draw a plan to the scale $\frac{1}{8}$.

3. If a line 1 in. long in a drawing represents a distance of 8 ft., what distance is represented by a line $3\frac{3}{4}$ in. long? by a line $4\frac{3}{8}$ in. long? by a line 1.5 in. long?

4. If the scale is 1 in. to 1 ft., what distance on a drawing will represent 6 ft. 3 in. in the object drawn?

5. A drawing of a rectangular floor 20 ft. by 28 ft. is 5 in. by 7 in. What scale was used?

6. A farmer plotted his farm as here shown, using the scale of 1 in. to 40 rd. Find the dimensions of each plot.



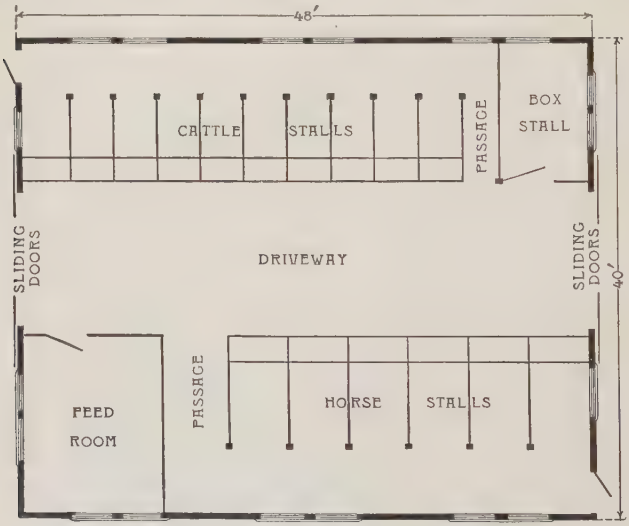
7. A plan of a rectangular school garden is drawn to the scale of 1 in. to 2 ft. 6 in. The plan is 18 in. long and $12\frac{1}{2}$ in. wide. What are the dimensions of the garden?

8. The infield of a baseball diamond is 90 ft. square. Draw a plan to the scale of 1 in. to 20 ft.

9. The field of play of a football field is 300 ft. long and 160 ft. wide. Lines parallel to the ends of the field are drawn at intervals of 5 yd., and the goals, 18 ft. 6 in. wide, are placed at the middle of the ends of the field. Draw a plan to the scale of 1 in. to 60 ft. and indicate the position of the goals and of the 5-yard lines.

10. A double tennis court is 78 ft. long and 36 ft. wide. Lines are drawn parallel to the longer sides and 4 ft. 6 in. from them, and the service lines are parallel to the ends and 18 ft. from them. The net is halfway between the ends. Draw a plan to any convenient scale.

11. The drawing here shown is the floor plan of a certain type of barn. Determine the scale to which the

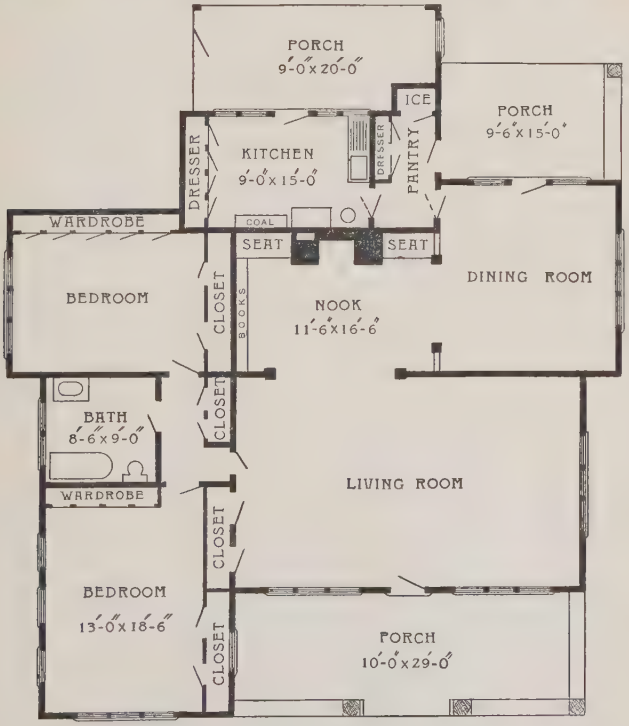


plan is drawn, find the width of the driveway in the barn, the width of each horse stall, the width of each cattle stall, and the dimensions of the box stall and the feed room.

12. A class in domestic science drew a plan for a model kitchen in an apartment house, using the scale $\frac{1}{60}$. If the plan is 3 in. long and 2 in. wide, what are the actual dimensions of the kitchen?

13. A drawing was made of a lamp screen $20\frac{1}{2}$ in. high. The drawing being $2\frac{9}{16}$ in. high, what scale was used?

14. The drawing below is the plan for a concrete bungalow. Find the scale used in drawing the plan.



15. In Ex. 14 find the dimensions of the living room, dining room, and smaller bedroom including wardrobe.

JM1

Accurate Proportions. Suppose that you measure a rectangular room and find it to be 20 ft. long and 16 ft. wide, and suppose that you measure a drawing of the room and find it to be 10 in. long and 6 in. wide. You would conclude that the drawing is not a good one, because the width should be, as in the room, $\frac{4}{5}$ of the length.

An accurate drawing or picture must maintain the proportions of the object.

That is, if the width of the object is $\frac{4}{5}$ of the length, the width of the object shown in the drawing must be $\frac{4}{5}$ of the length in the drawing; if the width of the object is $\frac{7}{8}$ of the length in one case, it must be $\frac{7}{8}$ of the length in the other case; and so on for other proportions.

It is better at this time to explain informally the meaning of proportion, as is done above. A more formal explanation of the subject of proportion is given later in the book when it is needed.

Exercise 11. Accurate Proportions

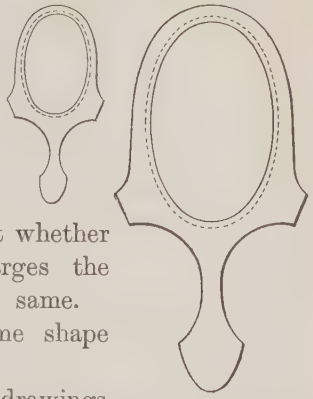
1. A house is 36 ft. high and the garage is 20 ft. high. If the house is represented in a drawing as 18 in. high, how high should the drawing of the garage be?

In all such cases the objects are supposed to be at approximately the same distance from the eye, so that the element of perspective does not enter.

2. A landscape gardener is drawing to scale a plan for a rectangular flower garden 18 ft. long and 14 ft. wide. In the drawing the length is represented by $6\frac{3}{4}$ in. By what should the width be represented?

3. Draw a right triangle whose sides are 3 in., 4 in., and 5 in. respectively, and draw another right triangle of the same shape but with the hypotenuse $1\frac{1}{4}$ in. long.

Similarity of Shape. As we have already seen, it is frequently necessary to draw a figure of the same shape as another one, but not of the same size. For example, an architect or a map drawer may reduce the original by using a small scale, but if we are making a drawing of a small object seen through a microscope we use a large scale. But whether the drawing reduces or enlarges the original, the shape remains the same.



Figures which have the same shape are said to be *similar.*

For example, here are two drawings of a hand mirror. In outline each drawing is similar to the mirror itself, and each is also similar to the other.

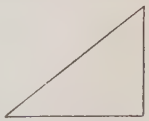
Figures which are similar to the same figure are similar to each other.

Two maps of a state are not only similar in outline to the state itself, but each is similar in outline to the other.

Exercise 12. Similarity of Shape

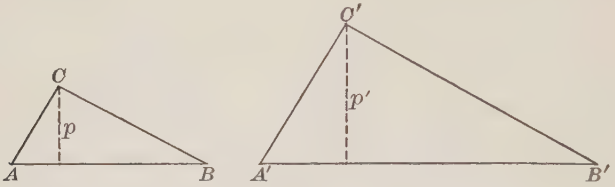
1. Construct three equilateral triangles whose sides are respectively 2 in., $3\frac{1}{2}$ in., and 5 in. Are they similar?

2. Construct three rectangles, the first being $1\frac{1}{2}$ in. by $2\frac{1}{2}$ in.; the second, 3 in. by 5 in.; and the third, 2 in. by $2\frac{1}{2}$ in. If they are not all of the same shape, discuss the exception.



3. Construct a right triangle of the same shape as this triangle but twice as high, and another of the same shape but three times as high.

Angles in Similar Figures. Here are two similar right triangles, ABC and $A'B'C'$, and in each triangle a perpendicular (p, p' respectively) is drawn from the vertex of the right angle to the hypotenuse.



Are the figures still similar? Are the sides proportional? What can be said as to the corresponding angles?

This brings us to another property of two similar figures, namely, that the angles of one are equal respectively to the angles of the other. That is, *in similar figures, corresponding lines are in proportion and corresponding angles are equal.*

A close approximation to similar figures may be seen in the case of moving pictures. The large picture shown on the screen is substantially similar to the small picture on the reel, although there is some distortion, particularly around the edges.

Exercise 13. Similar Figures

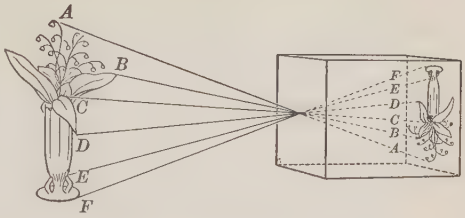
All work oral

State which of the following pairs of figures are necessarily similar and state briefly the reasons in each case:

1. Two squares.
2. Two triangles.
3. Two circles.
4. Two rectangles.
5. Two isosceles triangles.
6. Two equilateral triangles.

7. State whether two parallelograms, each side of one being 3 in. and each side of the other being 4 in., must always be similar, and give the reason for your answer.

Similar Figures in Photographs. If you have ever used a plate camera you have seen that there is a piece of ground glass in the back and that an object in front of the camera appears inverted on this ground glass. The reason is clear, for the ray of light from the point A of the flower passes through the lens of the camera and strikes the plate at A. That is,



On a photographic plate the figure is similar in outline to the original, but is inverted.

There is, of course, a slight distortion on account of the refraction of the rays of light in passing through the lens.

If the camera is 8 in. long and the object is 16 in. away from the lens O, an object 5 in. high will appear as $2\frac{1}{2}$ in. high on the plate. That is, since the length of the camera, $B'O$, is half the distance of the object from O, or half of OB, we see that $A'B'$, the height of the object on the plate, is half of AB, that is, half the real height.

Similarly, if the length of the camera is 10 in., and the height of an object 18 ft. away is 5 ft., we can easily find the height of the object on the plate as follows:

Reducing all the measurements to inches, we have

$$18 \text{ ft.} = 18 \times 12 \text{ in., and } 5 \text{ ft.} = 5 \times 12 \text{ in.}$$

Then
$$\frac{10}{18 \times 12} \times 5 \times 12 \text{ in.} = 2\tfrac{7}{9} \text{ in.}$$

The teacher is advised to solve this on the blackboard.

Exercise 14. Similar Figures in Photographs

1. A man 5 ft. 8 in. tall stands 16 ft. from a camera which is 8 in. long. What will be the height of his photograph? Explain by drawing to scale.

2. The photograph of a man who is 5 ft. 8 in. tall is 6 in. high, and the camera is 10 in. long. How far did the man stand from the camera?

3. If a boy's face is 8 ft. from a camera which is 10 in. long, the height of the photograph of his face is what proportion to the height of his face? If he places his hand 2 ft. nearer the camera, the length of the photograph of his hand is what proportion to the length of his hand?

One of the first things a beginner has to learn in using a camera is that objects appear distorted unless they are at about the same distance from the camera, especially if they are relatively near to it.

4. A tree photographed by a 4-inch camera at a distance of 10 ft. appears on the photograph as 6 in. high. How high is the tree?

We see by this problem that heights and distances can often be found by photography; and, in fact, much difficult engineering work is now done with the aid of photographs.

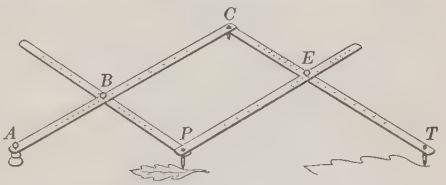
5. A camera is held directly in front of the middle of a door and at a distance of 8 ft. from it. The door is 4 ft. by 7 ft. 6 in. and the length of the camera is 8 in. Find the dimensions of the door in the photograph.

6. A 10-inch camera is placed at a certain distance from a tree which is 50 ft. high, and a boy 5 ft. tall stands between the tree and the camera. The height of the boy in the photograph is $1\frac{1}{2}$ in., and the height of the tree 8 in. Find the distance of both the boy and the tree from the camera.

The Pantograph. Probably you have seen an instrument which is extensively used by architects, draftsmen, designers, and map makers in drawing plane figures similar to other plane figures. It usually consists of four bars parallel in pairs, and is known as a *pantograph.*

In explaining the pantograph it becomes necessary to speak of the *ratio* of two lines. By the ratio of 2 ft. to 5 ft. is meant the quotient $2\text{ ft.} \div 5\text{ ft.}$, or $\frac{2}{5}$, and by the ratio of $\frac{1}{2}$ in. to $3\frac{1}{4}$ in. is meant $\frac{1}{2}\text{ in.} \div 3\frac{1}{4}\text{ in.}$, or $\frac{2}{13}$. Likewise, by the ratio of a line AB to a line CD is meant the quotient found by dividing the length of the line AB by the length of CD. This ratio is written AB/CD, or $AB:CD$. If AB is half CD, then $AB:CD=\frac{1}{2}$. This is read "the ratio of AB to CD is equal to one half."

In the figure the bars are adjustable at B and E. The end A is fixed, that is, it remains in the same place while the pantograph is being used. A tracing point is placed at T and a pencil at P, and BP and PE are so adjusted as to form a parallelogram $PECB$ such that any required ratio $AB:AC$ is equal to $CE:CT$. Then as the tracer T traces a given figure, the pencil P draws a similar figure. If the given figure is to be enlarged instead of reduced, the pencil and the tracing point are interchanged.



This discussion of the pantograph has little value unless the instrument is actually used by the students. A fairly good one can be made of heavy cardboard or of strips of wood, and school-supply houses will furnish a school with the instrument at a low cost.

A simple pantograph can be made by fastening a rubber elastic at one end, sticking a pencil point through the other end, and placing a pin for a tracer anywhere along the band.

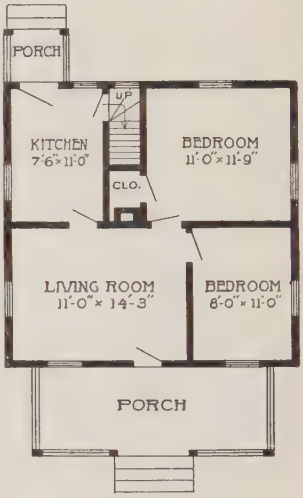
Exercise 15. The Pantograph

1. Draw a plan of your schoolroom to scale and then enlarge it to twice the size with the aid of a pantograph.

This exercise should be omitted in case the school is not supplied with a pantograph.

2. Find the map of your state in a geography and reduce it to half the size by using a pantograph.

3. By using a pantograph reduce the size of this plan of a cottage to two thirds its present size.



FIRST FLOOR PLAN

This can be done by laying this page flat on the drawing board while someone holds the book. It is better, however, to copy the plan on paper and use the pantograph with the drawing. It is desired that the student should use the pantograph a few times in connection with various kinds of work in which it is really used in practical life.

4. By using a pantograph enlarge this sketch for a child's coat to three times the given size.

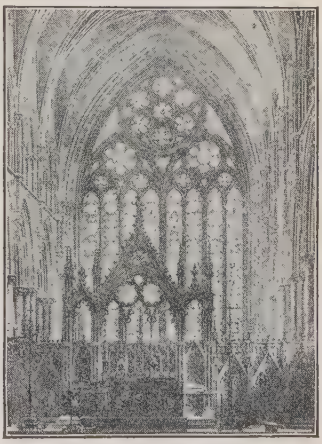
In addition to this, other similar drawings should be made and then enlarged. Of late the pantograph has come into extensive use by dressmakers for the purpose of enlarging designs of this kind.

5. Draw a sketch of any object in the room and reduce the sketch to one third its size by using a pantograph.

6. Draw a sketch of a tree near the school and enlarge the sketch to five times its size by using a pantograph.

Symmetry. If we place a drop of ink on a piece of paper and at once fold the paper so as to spread the ink, we shall often find curious and interesting forms frequently resembling flowers, leaves, or butterflies. These forms are even more interesting if we use a drop of black ink and a drop of red ink. The interest in such figures comes from the fact that they are *symmetric*, that is, that one side is exactly like the other.

In this case we say that the figure is *symmetric with respect to an axis*, this axis being the crease in the paper or, more generally, the line which divides the figure into two parts that will fit each other if folded over.

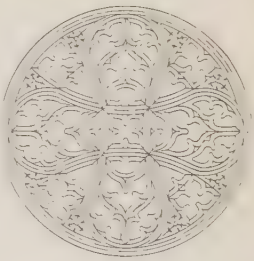


In architecture we often find symmetry with respect to an axis. For example, in this picture of the interior of a great cathedral we see that much of the beauty and grandeur is due to symmetry.

This case is evidently one of *symmetry with respect to a plane* instead of with respect to a line. We may also have *symmetry with respect to a center*, that is, a figure may turn halfway round a point and appear exactly as at first. This is seen in a circle, or, among solids, in a sphere. It is also seen in the Gothic window shown on page 148. Symmetry of all kinds plays a very important part in art, not merely in architecture, painting, and sculpture, but in all kinds of decoration.

Exercise 16. Symmetry

1. Has this Gothic window an axis of symmetry? If so, draw the circle and indicate the axis of symmetry. If it has more than one axis of symmetry, draw each axis of symmetry.



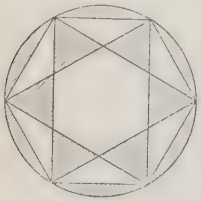
2. If the figure has a center of symmetry, indicate this center in your rough sketch by the letter *O*.

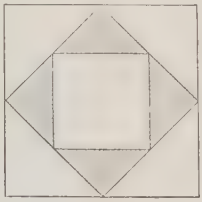
3. Draw an equilateral triangle and draw all its axes of symmetry.

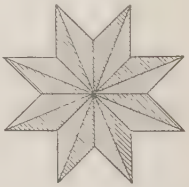
4. Draw a square and draw all its axes of symmetry.

5. Draw a plane figure with no axis of symmetry: one having only one axis of symmetry: one having two axes of symmetry; one having any number of axes of symmetry.

6. Draw the following designs in outline and indicate by letters all the axes of symmetry in each design:



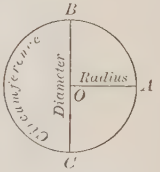




7. Write a list of three windows in churches in your locality which have axes of symmetry. If you know of any window which has a center of symmetry, mention it.

The class should be asked to mention other illustrations of axes of symmetry, as in doors and in linoleum patterns. There should also be questions concerning planes of symmetry, as in a cube, a sphere, a chair, animals, and vases. Objects in the schoolroom offer a good field for inquiry.

Plane Figures formed by Curves. We have already mentioned a number of figures formed by curve lines without attempting to define them. We shall now mention these again and shall discuss more fully a few of those which occur most frequently in drawing, pattern making, architecture, measuring, and the like.

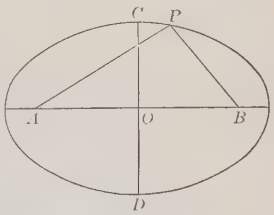


This figure represents a *circle* with *center O*, *radius OA*, and *diameter BC*.

The circle is sometimes thought of as the space inclosed and sometimes as the curve line inclosing the space. The length of this curve is called the *circumference*, and sometimes the curve itself is called by this name.

It is not expected that the above statement will be considered as a formal definition to be learned. All that is needed at this time is that the terms shall be used properly. Teachers should recognize that *circle* and *circumference* both have two meanings, as stated above.

Another interesting figure, but one which is used not nearly so often as the circle, is the *ellipse*. If we place two thumb tacks at *A* and *B*, say 3 in. apart, and fasten to them the ends of a string which is more than 3 in. long, draw the string taut with a pencil point *P*, and then draw the pencil round while keeping the string taut, we shall trace the ellipse.

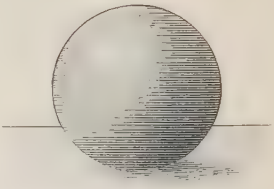


It is evident that an ellipse has two axes of symmetry and one center of symmetry.

The orbits of the planets about the sun are ellipses.

When facilities for drawing permit, the student should draw ellipses of various sizes and shapes and should satisfy himself that two ellipses are not in general similar.

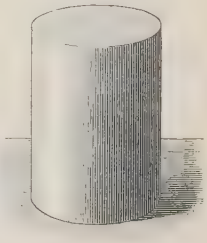
Solids bounded by Curved Surfaces. We have often mentioned the sphere, and shall now speak of it and of other solids bounded in whole or in part by curved surfaces.



A *sphere* is a solid bounded by a surface whose every point is equidistant from a point within, called the *center*.

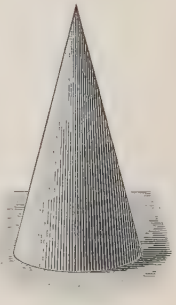
We also speak of the *radius* and *diameter* of a sphere, just as we speak of the radius and diameter of a circle.

A *cylinder* is a solid bounded by two equal circles and a curved surface as shown in this figure.



The two circles which form the ends are called the *bases* of the cylinder and the radius and diameter of either base are called respectively the *radius* and *diameter* of the cylinder.

The line joining the centers of the two bases is called the *axis* of the cylinder, and its length is called the *height* or *altitude* of the cylinder.



A *cone* is a solid like the one here shown. It has a circular base and an axis of symmetry from the center of the *base* to the *vertex* of the cone. The perpendicular distance from the vertex to the base is called the *height* or *altitude* of the cone. The words *radius* and *diameter* are used as with the cylinder.

In this book we shall consider only cylinders and cones in which the axes are perpendicular to the bases.

Exercise 17. Solids bounded by Curves

1. If you cut off a portion of a sphere, say a wooden ball, by sawing directly through it, but not necessarily through the center, what is the shape of the flat section? Illustrate by a drawing.

2. In Ex. 1 is the section always a plane of symmetry? If not, is it ever a plane of symmetry, and if so, when?

3. A cylinder is symmetric with respect to what line or lines? with respect to what plane or planes?

4. Could a cylindric piece of wood, say a broom handle, be so cut that the section would be a circle? If so, how should it be cut? Could it be so cut that the section would seem to be an ellipse? Illustrate each answer.

The section last mentioned is really an ellipse, and this is proved in higher mathematics.

5. Could a cylindric piece of wood be so cut that the section would be a rectangle? a trapezoid? Illustrate.

6. Three cylinders of the same height, 4 in., have as diameters 3 in., 4 in., and 5 in. respectively. Can a section in any one of them be a square? Illustrate the answer.

7. Is a cone symmetric with respect to any line? to any plane? Illustrate each answer.

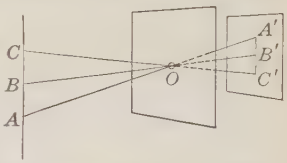
8. How could a cone be cut so as to have the section a circle? a triangle? apparently an ellipse? Illustrate.

The section last mentioned is really an ellipse, and this is proved in higher mathematics. This is the reason why an ellipse is called a conic section. Other conic sections are studied in higher mathematics, and they are important in the study of astronomy, mechanics, and other sciences.

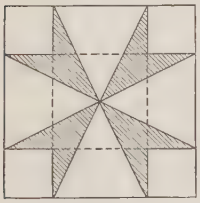
9. How is the largest triangle obtained by cutting a cone? Illustrate the answer.

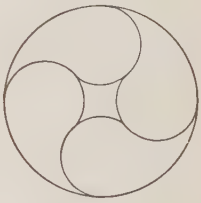
Exercise 18. Review

1. If the rays of light from any object ABC pass through a small aperture O of an opaque screen and fall upon another screen parallel to the object, an inverted image $A'B'C'$ will be formed as here shown. If the object is 5 ft. long and 9 ft. from O, how far from O must the second screen be placed so that the image shall be 6 in. long? How far, so that the image shall be 8 in. long?



2. With the aid of ruler and compasses, construct figures similar to each of the following figures, but twice as large, and indicate the axis, axes, or center of symmetry of each.





3. Draw the following figure about half as large again and make it the basis for a pattern for linoleum, using other lines as necessary.

4. Draw a square and cut it into four triangles by means of two diagonals. Describe the triangles with respect to their being equilateral, isosceles, or right.

5. Draw this figure about half as large again and make it the basis for a pattern for a church window, using other lines as may be necessary for the purpose.

Exercise 19. Optional Outdoor Work

1. Collect, if possible, several leaves of each of the following kinds of tree: oak, elm, maple, pine, and poplar. Are the leaves of each kind of tree approximately similar to the other leaves of the same tree? Has each of the leaves an axis of symmetry?

2. Do you know of any building lots or fields that are triangular? If so, make rough outline drawings of them.

3. Do any of the public buildings of your community have cylindric columns? If so, which buildings?

4. Do you know of any church spires that are conic in shape? If so, which spires?

5. What is the shape most frequently used in decorating the interiors of churches in your vicinity?

6. Can you find an illustration of a Gothic window in any of the churches in your vicinity? If so, where?

7. If there is a standpipe in your vicinity, what is its shape? What is the shape of most of the smokestacks of the factories in your community?

8. Notice the designs in the carpeting, wall paper, and linoleum exhibited by various stores. What general pattern or design is most frequently used?

9. If convenient, inspect a house that is being built and compare the floor plan with the plans of the contractor or architect. What scale was used in drawing the plan?

10. Name illustrations of each of the following forms that you have seen in your community: circle, rectangle, cylinder, cone, sphere, trapezoid, and triangle.

As stated above, this work is purely optional. It is suggestive of a valuable line of local questions.

Exercise 20. Problems without Figures

1. How do you construct a triangle, having given the lengths of the three sides?

2. How do you construct an isosceles triangle, having given the base and one of the equal sides?

3. How do you construct an equilateral triangle, having given one of the sides?

4. State two methods of drawing from a given point a perpendicular to a given line.

5. If you have a line drawn on paper, what is the best way you know to bisect it?

6. How do you bisect an angle?

7. How do you construct an angle exactly equal to a given angle?

8. How do you draw a line parallel to a given line?

9. If you have a line drawn on paper and wish to divide it into five equal parts, how do you proceed?

10. How do you construct a six-sided figure in a circle, the sides all being equal?

11. How do you draw to a given scale the rectangular outline of the printed part of this page?

12. How do you draw a plan of the top of your desk to the scale of a certain number of inches to a foot?

13. When you know the scale which was used in drawing a map, how do you find the actual distance between two cities which are shown on the map?

14. How can you enlarge a drawing by the aid of a pantograph?

15. How do you determine whether a figure is symmetric with respect to an axis?

II. GEOMETRY OF SIZE

Size. On page 111 we found that geometry is concerned with three questions about any object: What is its shape? How large is it? Where is it? Thus far we have considered the shape of objects; we shall now consider size.

There are several ideas to be considered when we think and speak of the size of objects, such as length, area, and volume, all of which we may include in the single expression *geometric measurement.* That is, we shall not think of size as including the measurement of weight, of value, of hardness, and the like, but only as including the length (width, height, depth, and distance in general), area (surface), and volume (capacity) of figures.

Length. It seems very easy to measure accurately the length of anything, but it is not so easy as it seems. Linen tape lines stretch, steel tape lines contract in cold weather, ordinary wooden rulers shrink a little when they get very dry, and chains wear at the links and thus become longer with age. But these matters are of less moment than the carelessness of those who make the measurements. If the members of your class, each by himself, should measure the length of the walk in front of your school, to the nearest sixteenth of an inch, and not compare results until they had finished, it is likely that each would have a different result. In fact, *all measurement is simply a close approximation.*

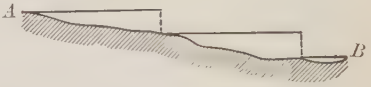
One of the best ways of securing a close approximation to the true result is to make the measurement in two different ways. *Never fail to check a measurement.*

Just as we should always check an addition by adding in the opposite direction, so we should always check a measurement of length by measuring, if possible, in the opposite direction.

Outdoor Work. In connection with the study of the size and position of common forms we shall first suggest a certain amount of work to be done out of doors.

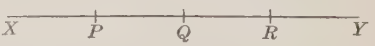
1. Measure the length of the school grounds.

To do this, drive two stakes at the appropriate corners, putting a cross on top of each stake so as to get two points between which to measure. Measure from A to B by holding the tape taut and level, drawing perpendiculars when necessary by means of a plumb line as shown in the figure. Check the work by measuring from B back to A in the same way.



2. Run a straight line along the sidewalk in front of the school yard.

Of course for a short distance this is easily done by stretching a string or a measuring tape, but for longer distances another plan is necessary.

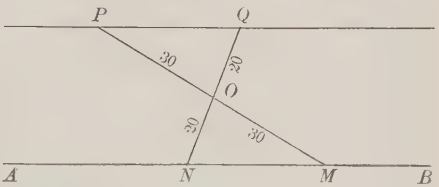


If we wish to run a line from X to Y, say 300 ft., we drive stakes at these points and mark a cross on the top of each so as to have exact points from which to work. Now have one student stand at X and another at Y, each with a plumb line marking the exact points. Then have a third student hold a plumb line at some point P, the student at X motioning him to move his plumb line to the right or to the left until it is exactly in line with X and Y. A stake is then driven at P, and the student at X moves on to the point P. The point Q is then located in the same way. In this manner we stake out or "range" the line from X to Y, checking the work by ranging back from Y to X.

3. Measure the height of a tree by making on the ground a right triangle congruent to a right triangle which has the tree as one side.

To do this, sight along an upright piece of cardboard so as to get the angle from the ground to the top of the tree. Mark the angle on the cardboard and then turn the cardboard down flat so as to have an equal angle on the ground. A right triangle can now easily be laid out on the ground so as to be congruent to the one of which the tree is one side. By measuring a certain side of this right triangle, the height of the tree can be found.

4. Run a line through a point P parallel to a given line AB for the purpose of laying out one of the two sides of a tennis court.



Stretch a tape line from P to any point M on AB, bisect the line PM at O, and from any point N on AB draw NO. Prolong NO to Q, making OQ equal to NO, and draw PQ. Suppose that ON is 20 ft. Then sight from N through O, and place a stake at Q just 20 ft. from O. Then P and Q determine a line parallel to AB.

The proof of this fact, like the proofs of many other facts inferred from certain of the exercises, is part of demonstrative geometry, which the student will meet later in his course in the high school.

Outdoor work will be given at intervals and always by itself, so that it can easily be omitted. The circumstances vary so much in different parts of the country as to climate, location of the school, and other conditions, that a textbook can merely suggest work of this kind which may or may not be done, as the teacher directs. A good tape line, three plumb lines (lines with a piece of lead at one end), and a pole about 10 ft. long will serve for an equipment for beginners.

Exercise 21. Practical Measurements of Length

1. Measure the length of this page to the nearest thirty-second of an inch, checking the work.

If a ruler is used which is, as usual, divided only to eighths of an inch, the student will have to use his judgment as to the nearest thirty-second of an inch. The protractor illustrated on page 115 has an edge on which lengths are given to sixteenths of an inch, and such a scale may be used if laid along the edge of a ruler.

2. Measure the length of this page to the nearest twentieth of an inch, checking the work.

The protractor illustrated on page 115 has an edge divided into tenths of an inch. There is advantage in the student becoming familiar with the units of the metric system, even before he studies the subject on page 205, since these units have come into use in our foreign trade and in all our school laboratories. It is desirable to know that 10 millimeters (mm.) = 1 centimeter (cm.) = 0.4 in., nearly; 10 cm. = 1 decimeter (dm.); 10 dm. = 1 meter (m.) = 39.37 in.

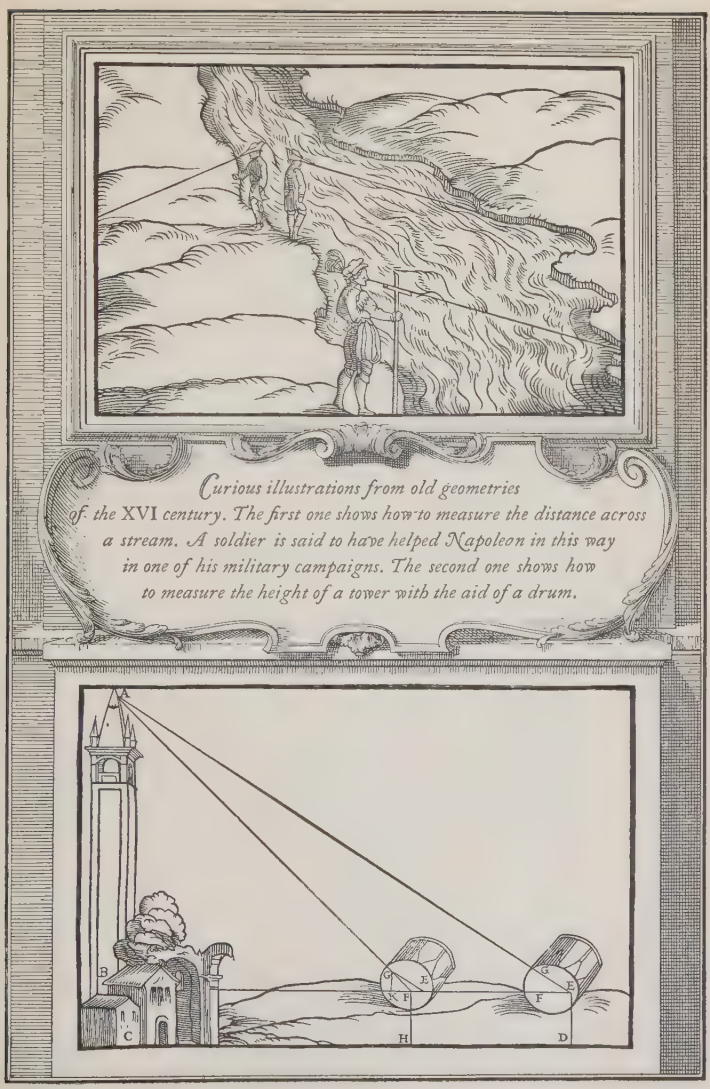
3. Measure the length of the longest line of print on this page, to the nearest sixteenth of an inch, checking the work.

If the student has a pair of dividers (compasses with sharp points), this may be used to transfer the length to a ruler. This method is usually more nearly accurate than to lay the ruler on the page.

4. Measure the length of your schoolroom to the nearest eighth of an inch, checking the work.

If the class works in groups of two, and each group checks its result with care, there may still be some difference. In that case an average may be taken. This is commonly done in surveying.

5. In the upper part of the opposite picture a man is sighting across the stream in line with the front part of his cap. He then turns and sights along the ground, as shown by the other man standing near him. How does he find the width of the stream by this method?



Curious illustrations from old geometries of the XVI *century. The first one shows how to measure the distance across a stream. A soldier is said to have helped Napoleon in this way in one of his military campaigns. The second one shows how to measure the height of a tower with the aid of a drum.*

6. Draw a line 6 in. long, and on it measure off 2.8 in. from one end and 2.3 in. from the other end. Check the results by measuring the length of the intermediate portion. What should that length be? What do you find it to be by actual measurement?

7. Draw a line 3.9 in. long, and on it measure off successive lengths of $1\frac{3}{16}$ in. and $1\frac{5}{8}$ in. Check the results by measuring the length of the remaining portion, as in Ex. 6. What should that length be? What do you find it to be by actual measurement?

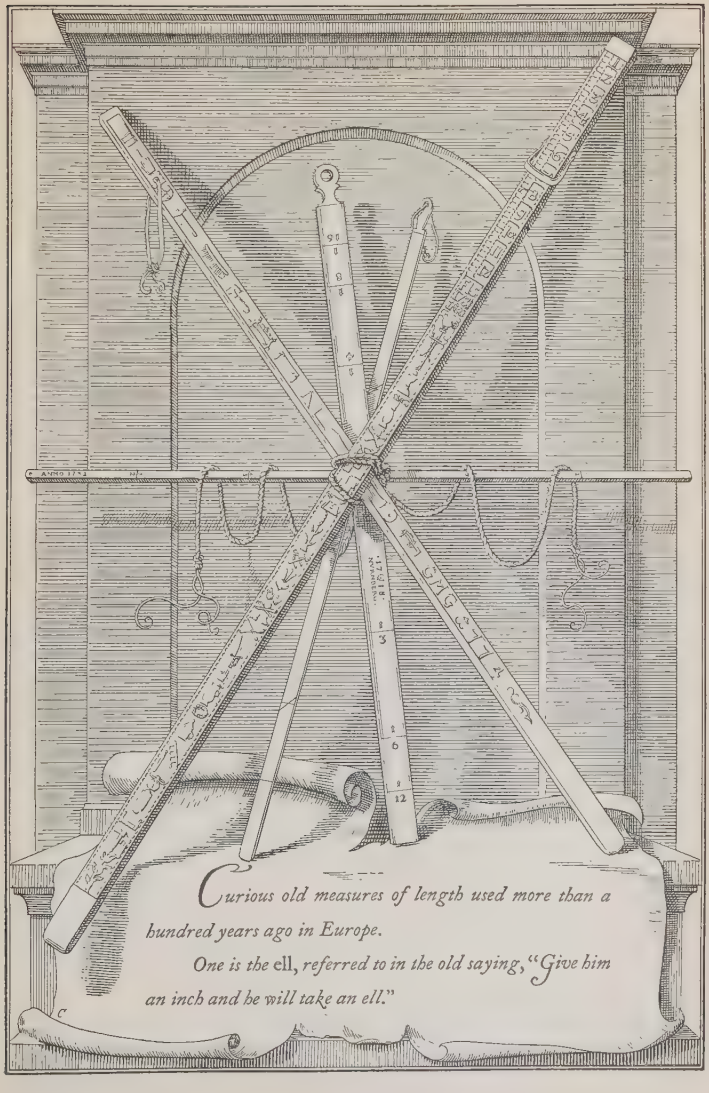
8. Draw a line $4\frac{5}{16}$ in. long, and on it measure off a line $2\frac{5}{32}$ in. long. Check the results by bisecting the original line as on page 124 and seeing if the point of bisection falls at the end of the part measured off.

9. Draw a line $9\frac{1}{2}$ in. long, and on it measure off successive equal lengths of $3\frac{1}{6}$ in. Check the results by dividing the original line into three equal parts by the first method given on page 130.

10. Construct a square 1 in. on a side by the methods already learned, and measure the length of each diagonal. What do you find it to be by actual measurement? If the square is accurately constructed, what must be the length of each diagonal to the nearest tenth of an inch; that is, what is the square root of 2?

11. Construct a rectangle 3 in. high and 4 in. long. Check the accuracy of the construction by measuring the length of each diagonal. What do you find it to be by actual measurement?

Such measurements give some idea of the accuracy required in a machine shop. With the instruments which the students have, these measurements are as nearly accurate as can be required, but for practical purposes much closer approximations are often necessary.



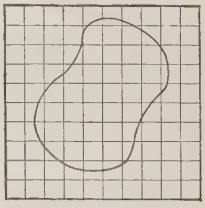
Curious old measures of length used more than a hundred years ago in Europe.

One is the ell, *referred to in the old saying, "Give him an inch and he will take an ell."*

Estimates of Area. There are several methods for estimating areas, of which we shall now consider two. In general it will be found that neither of these plans is very practical, although both are used in certain difficult cases of measurement. The most practical way of finding areas is introduced on page 164.

1. If an area inclosed by a curve is drawn on squared paper, the area may be estimated approximately by counting the squares contained within the curve. The squares on the boundary should be included or excluded according as more than half their area is included or is not included within the bounding line, and half the other squares that are practically half within and half without should be included. In certain cases the rule should be altered as the peculiarity of the case requires.

For example, in this figure if each square represents 1 sq. in., the area inclosed by this curve is approximately 33 sq. in., there being approximately 33 squares inclosed.



Paper such as that used in the illustration is called *squared paper* or *cross-section paper*. It can generally be bought at any stationer's. When the paper is ruled into squares one tenth of an inch on each side, there are, of course, 100 such squares in 1 sq. in.

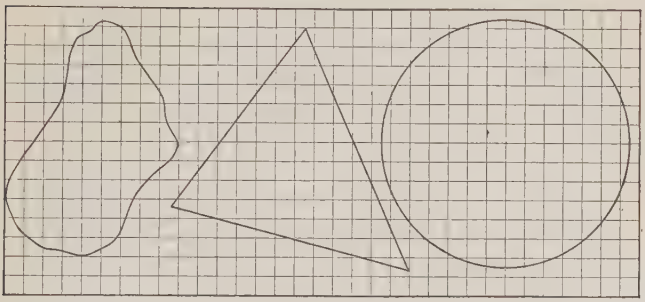
In case it is not easy to purchase squared paper, ruled in tenths of an inch, it is advisable for the student to rule some paper, drawing the lines with the same care taken in making the other constructions of geometry.

2. The area inclosed by a curve drawn on thick paper or cardboard may be estimated by cutting out the area to be measured, weighing it, and comparing its weight with that of a unit of area, such as a square inch cut from the paper.

Since this method considers weight, it is not geometric, and furthermore it is not very practical in estimating small areas.

Exercise 22. Estimates of Areas

1. Estimate the area of each of the following figures:



2. Draw the outline of a leaf on squared paper and estimate the area.

Paper often comes ruled in millimeters, in which case the areas can be found with greater accuracy, to square millimeters.

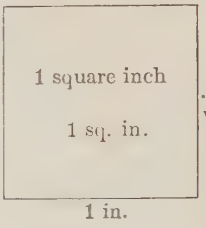
3. On a piece of squared paper draw a rectangle 2 in. long and 1.7 in. wide, and divide it into two triangles by drawing either diagonal. Estimate the area of each triangle, state whether the areas are equal, and check the work by finding the area of the rectangle and showing that this is equal to the sum of the areas of the triangles.

4. On a piece of squared paper draw a parallelogram 1.9 in. long and 1 in. high. Draw either diagonal, estimate the area of each triangle thus formed, and proceed further, as in Ex. 3.

5. On a piece of squared paper draw a trapezoid 1 in. high, with lower base 2 in. and upper base 1.2 in. Estimate the area of the trapezoid by the method of Ex. 3.

This method of approximation for estimating areas is sufficient for many purposes, but the methods of geometry, some of which we shall now study, are greatly superior to this method.

Unit of Area. We have seen how we may estimate an area to a fair degree of accuracy. Whether we estimate or actually measure, we commonly express the area by means of some unit square, such as the square inch.



A square inch is not the same as 1 in. square; 1 sq. in. is the area of a space that is 1 in. square. A circle may have this area.

There are also other units of area, such as the acre (160 sq. rd.).

Area of a Rectangle. A school building has a rectangular entrance hall 10 ft. long and 4 ft. wide, the floor being made of marble squares 1 ft. on a side. What is the easiest way of finding the area of the floor?

There are 10 squares in each row and there are 4 rows of squares. Since there are 4×10 squares, we have:

$$\text{Area} = 4 \times 10 \text{ sq. ft.} = 40 \text{ sq. ft.}$$

The area of a rectangle is equal to the product of the base and height.

This means that $10 \times 4 = 40$, the *number* of square feet.

We often express this statement by a *formula*, using A for area, b for base, and h for height, thus:

$$A = b \times h,$$

or, briefly,

$$A = bh.$$

The absence of a sign between letters in a formula indicates multiplication.

In this book rooms, boxes, fields, and the like are to be considered as rectangular unless the contrary is stated.

Exercise 23. Area of a Rectangle

Using the formula given on page 164, find the areas of the following rectangles:

1. $18\frac{1}{2}$ ft. by 26 ft.
2. 27 yd. by $42\frac{1}{2}$ yd.
3. $13\frac{1}{2}$ rd. by 28 rd.
4. $7\frac{1}{2}$ in. by $9\frac{3}{4}$ in.
5. 36.3 ft. by 142.5 ft.
6. 12.8 ft. by 17.3 ft.
7. 42.3 yd. by 46.8 yd.
8. $37\frac{1}{2}$ in. by 62 in.

9. Find the floor area of a room that is 28 ft. 6 in. long and 32 ft. wide.

10. Find the area of a sheet of paper $3\frac{1}{4}$ in. square. Verify the result by drawing the figure and ruling it off into $\frac{1}{4}$-inch squares.

11. A garden, 38 ft. by 56 ft., contains a 3-foot walk laid inside the garden along the four sides. The mid-points of the long sides are joined by a 2-foot path. Find the area left for cultivation, and draw a plan to scale.

12. On the floor of a room 32 ft. long and 24 ft. wide a border 2 ft. wide is to be painted. Find the cost of painting the border at 30¢ per square yard.

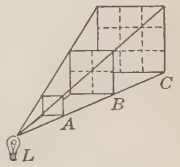
13. At 16¢ per square foot find the cost of cementing a walk 6 ft. wide round the outside of a garden that measures 56 ft. by 82 ft.

14. A student, being asked to measure a rectangle, understated the length 2% and overstated the width 3%. Find the per cent of error in the area computed.

15. Draw a plan of the floor of the basement of a house to scale as follows: Start at A, go north 14 ft. to B, west 8 ft. to C, north 5 ft. to D, west 8 ft. to E, south 19 ft. to F, and east to A. Find the cost of cementing the floor at 15¢ per square foot.

16. Construct a rectangle and then construct a similar rectangle the area of which is three times that of the first.

17. The screens A, B, and C are 1 ft., 2 ft., and 3 ft. respectively from an electric light L. If screen A should be removed, the quantity of light which fell on it would fall on B. If screens A and B should be removed, the same quantity of light would fall on screen C. How would the *intensity* of light compare for a given area on each of the screens?



18. How many paving blocks each 4 in. by 4 in. by 10 in., placed on their sides, will be required to pave a street 1800 ft. long and 34 ft. 8 in. wide?

19. Printers usually cut business cards from sheets 22 in. by 28 in. How many cards 2 in. by 3 in. can be cut from one sheet and how much will be wasted? Draw a plan.

20. At $6¢$ a square foot find the cost of enough wire screen for the 8 windows of a gymnasium, each being 28 in. by 64 in. inside the frame. Allowance should be made for the wire to overlap the frames 1 in. on every side.

21. Find the area of a double tennis court.

A standard double tennis court is 36 ft. by 78 ft.

22. What is the meaning of the statement $A = bh$?

23. How many acres are there in a football field?

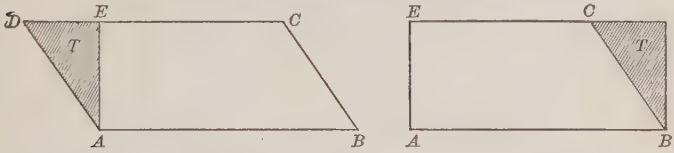
A standard football field is 100 yd. by 53 yd. 1 ft.

An acre is 160 sq. rd., and a rod is $5\frac{1}{2}$ yd. The teacher should give plenty of practical work in finding the areas of floors and the like.

24. A farm team plowing a field walks at the rate of 2 mi. per hour and is actually plowing $\frac{2}{3}$ of the time. What area will be plowed from 7 A.M. to noon if a plow is used which turns a furrow 14 in. wide?

Area of a Parallelogram. It is often convenient or necessary to find the area of a parallelogram.

If from any parallelogram, like $ABCD$ in the first figure, we cut off the shaded triangle T by a line perpendicular



to DC, and place the triangle at the other end of the parallelogram, as shown in the figure at the right, the resulting figure is a rectangle.

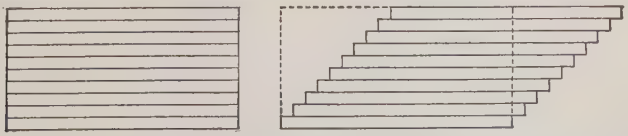
That is, the area of a parallelogram is equal to the area of a rectangle of the same base and the same height. But the formula for the area of a rectangle is $A = bh$.

Therefore *the area of a parallelogram is equal to the product of the base and height.*

This may be expressed by the formula

$$A = bh.$$

The teacher should make sure that the students understand the meaning of this formula. The purpose is to introduce algebraic forms as needed. The students should see that the value of A depends upon the values of b and h. In the language of more advanced mathematics, A is called a *function* of b and h. The students should see that all formulas are expressions of functions.



Rectangular pieces of cardboard, as in the figures shown just above, may be arranged to lead the student to infer that when the base and height of a rectangle are equal respectively to the base and height of a parallelogram, the areas are equal.

Exercise 24. Area of a Parallelogram

1. Draw parallelograms of the shapes and sizes of the following and show, by cutting off triangles and placing them as explained on page 167, that each parallelogram can be transformed into a rectangle of the same area.

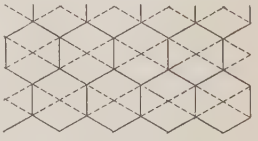
2. On squared paper draw four parallelograms and a rectangle, all having equal bases and equal heights, but all of different shapes. By cutting off a triangle from each parallelogram and moving it to the other side, transform each into a rectangle of the same area. Count the squares and compare the areas of the resulting rectangles.

Draw the following parallelograms to scale and find the area of each:

3. Base 30 in.; other side, 20 in.; height, 15 in.; scale $\frac{1}{10}$.

4. Base 12 in.; other side, 8 in.; height, 6 in.; scale $\frac{1}{2}$.

5. On squared paper draw a rectangle and a parallelogram with equal bases and equal heights. Compute the area of each by counting the included squares, and thus compare the areas.

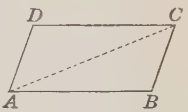


6. A floor is paved with six-sided tiles, as here shown. The tiles have been divided by dotted lines in the picture to suggest a method of measuring them. What measurements would you take to find the area of each tile? What other divisions of the tiles can you suggest for convenience in finding the area of each?

Exercise 25. Area of a Triangle

1. How is the area of a parallelogram found?

2. In the parallelogram here shown how do the areas of the triangles ABC and CDA compare? A triangle is what part of a parallelogram of the same base and height?



The parallelogram should be cut out of paper and then divided into two congruent triangles by cutting along one of the diagonals.

3. If the parallelogram in Ex. 2 is 6 in. wide and 3 in. high, what is its area? What is the area of each of the triangles formed by drawing the diagonal AC?

4. If a parallelogram is 5 ft. wide and 2 ft. high, what is its area? What is the area of each of the triangles?

5. If a parallelogram is 8 yd. wide and 3 yd. high, what is its area? What is the area of each of the triangles?

6. Find the area of a rectangle with base 7 ft. and height 10 ft.; of a triangle with base 7 ft. and height 10 ft.

Notice that a rectangle is one kind of a parallelogram.

7. Considering the above examples, state a rule for finding the area of a triangle.

8. Draw to scale a triangle with sides 6 in., 7 in., and 8 in. respectively. Draw lines to show that the area of the triangle is half the area of a rectangle with the same base and height.

9. What is the area of a triangular garden with base 32 ft. and height 16 ft.?

Find the areas of triangles with bases and heights as follows:

10. 4 in., 3.6 in.
11. 9 in., 7.4 in.
12. 8 yd., 9 yd.
13. 7.5 in., 8.4 in.
14. 9 ft., 4 ft. 4 in.
15. 6 ft. 3 in., 8 ft.

Area of a Triangle. From the illustrations given and the questions asked on page 169 it is easily seen that

The area of a triangle is equal to half the product of the base and height.

This may be expressed by the formula

$$A = \tfrac{1}{2}\, bh.$$

For example, what is the area of a triangle of base 14 in. and height 9 in.?

$$\tfrac{1}{2} \text{ of } 14 \times 9 \text{ sq. in.} = 63 \text{ sq. in.}$$

Exercise 26. Area of a Triangle

Examples 1 to 9, oral

State the areas of triangles with these bases and heights:

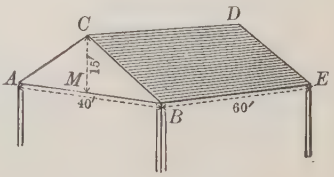
1. 12 in., 9 in.
2. 14 in., 11 in.
3. 9 in., 10 in.
4. 28 in., 3 in.
5. 8 in., 4.5 in.
6. 3.5 in., 6 in.
7. 32 in., 8 in.
8. 3.5 in., 4 in.
9. 8 in., 9.5 in.

10. What fraction of a square yard of bunting is there in a triangular pennant of base 56 in. and height 2 yd.?

Find the areas of triangles with these bases and heights:

11. 17 ft., 46 ft.
12. 19.5 ft., 18.3 ft.
13. 22.7 ft., 16.4 ft.
14. $36\tfrac{1}{2}$ ft., 17.6 ft.
15. 18.3 ft., 14.4 ft.
16. 29.7 yd., 24.8 yd.

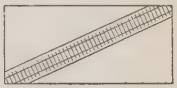
17. The span AB of a roof is 40 ft., the rise MC is 15 ft., the slope CB is 25 ft., and the length BE is 60 ft. Find the area of each gable end and the area of the roof.



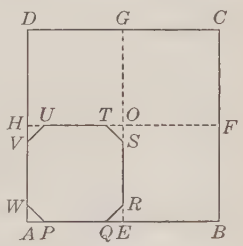
18. On squared paper draw a right triangle with the two sides respectively 1.5 in. and 2.5 in. Estimate the area by counting the squares, compute the area accurately, and then find what per cent the first result is of the second.

When we speak of the two sides of a right triangle we always mean the two perpendicular sides.

19. A field 65 rd. by 140 rd. is cut by a diagonal into two equal right triangles. A railway runs along this diagonal and takes 3 A. off each triangular field. How much is the rest of the field worth at \$140 an acre?



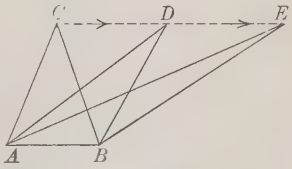
20. In this figure $ABCD$ represents an 8-inch square, E, F, G, and H being the mid-points of the sides. In the square $AEOH$, $AP = QE = ER = SO = OT = \cdots = \frac{1}{6} AE$. Find the area of each of the small triangles and also of the octagon $PQRSTUVW$.



The dots ($\cdots$) mean "and so on" and, in this case, take the place of "$UH = HV = WA$."

An *octagon* is a figure of eight sides.

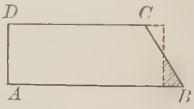
21. The triangle ABC is made by driving pins at A and B, running a rubber band around them, and stretching this band to the point C. Now imagine C to move along CE parallel to AB, stopping first at D and then at E. Have ABC, ABD, and ABE different areas? State your reasons fully.



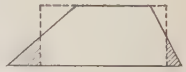
Since any field may be cut into triangles by drawing certain diagonals, the students are now prepared to find the area of any piece of land that admits of easy measurement.

Area of a Trapezoid. If a trapezoid T has its double cut from paper and turned over and fitted to it, like D, the two together make a parallelogram. How does the area of the whole parallelogram compare with the area of the trapezoid T? How does the base of the parallelogram compare with the sum of the upper and lower bases of the trapezoid? How do you find the area of the parallelogram? Then how do you find the area of the trapezoid?

If from the trapezoid $ABCD$, here shown, the shaded portion is cut off and is fitted into the space marked by the dotted lines, what kind of figure is formed? How is the area of the resulting figure found?



If the shaded portions of this trapezoid are fitted into the spaces marked by the dotted lines, what kind of figure is formed? How is the area of the resulting figure found?



From these illustrations we infer the following:

To find the area of a trapezoid, multiply the sum of the parallel sides by one half the height.

This may be expressed by the formula

$$A = \tfrac{1}{2}h(B + b),$$

where A stands for the area, h for the height, B for the lower base, and b for the upper base.

The parentheses show that B and b are to be added before the sum is multiplied by $\frac{1}{2}h$.

For example, if $h = 4$, $B = 7$, and $b = 5$, we have

$$A = \tfrac{1}{2} \times 4 \times (7 + 5)$$
$$= 2 \times 12 = 24.$$

Exercise 27. Area of a Trapezoid

Examples 1 to 6, oral

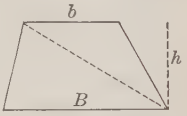
Find the area of each of the trapezoids whose height is first given below, followed by the parallel sides:

1. 6 in.; 9 in., 11 in.
2. 8 in.; 14 in., 6 in.
3. 12 in.; $4\frac{1}{2}$ in., $3\frac{1}{2}$ in.
4. 11 in.; 8 in., 12 in.
5. 9 in.; $4\frac{1}{2}$ in., $5\frac{1}{2}$ in.
6. 13 in.; 11 in., 7 in.
7. 18 in.; 9.5 in., 27.3 in.
8. 24 in.; $11\frac{1}{2}$ in., $9\frac{3}{4}$ in.
9. 17 in.; 18 in., 26 in.
10. 14 ft.; 6 ft. 4 in., 9 ft.
11. 42 yd.; $19\frac{1}{2}$ yd., $37\frac{1}{2}$ yd.
12. 127 ft.; $96\frac{3}{4}$ ft., $108\frac{1}{4}$ ft.

13. Find the number of acres in a field in the form of a trapezoid, the parallel sides being $33\frac{1}{4}$ rd. and $17\frac{3}{4}$ rd. and the distance between these parallel sides being 14 rd.

14. If the area of a trapezoid is 396 sq. in. and the parallel sides are 19 in. and 17 in., what is the height?

15. In this figure show that we may find the area of the trapezoid by adding the areas of two triangles.



This should be taken up at the blackboard. The teacher should show that in this case we have $\frac{1}{2}hB + \frac{1}{2}hb = \frac{1}{2}h(B + b)$, just as

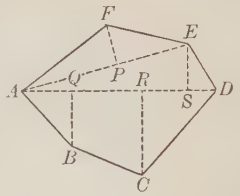
$$3 \times 2 + 3 \times 7 = 3 \times (2 + 7) = 3 \times 9.$$

A little algebra may thus be introduced as necessity requires and the way made easier for more elaborate algebra later.

16. Suppose that the upper and lower bases of a trapezoid are equal, does the formula for the trapezoid still hold true? The trapezoid becomes what kind of a polygon? The formula becomes the formula for what figure?

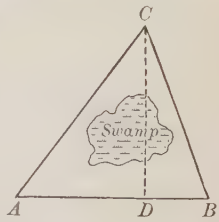
Practical outdoor work in measuring fields and in computing areas may now be given, or it may be postponed until after page 174 has been studied.

Area of any Polygon. A polygon like $ABCDEF$ may be divided into triangles, parallelograms, and trapezoids as here shown, and the areas of these parts may be found separately and then added.



As an exercise the teacher may assign to the class the finding of the area of the field here represented, the figure being drawn to the scale 1 in. = 200 rd.

Area found from Drawing. Suppose that the area of a field ABC has to be found, and that there is a large swamp as indicated in the figure. In such a case it is not easy to find the height of the triangle; that is, the distance CD. The lengths of the three sides may, however, be measured, and then the area may be found by drawing the outline to scale and measuring the height of this triangle.



Only the drawing to scale is here shown. If the scale is 1 in. = 100 rd., we see that CD is 90 rd., because CD is 0.9 in. If AB is 100 rd. the area of the triangle is $\frac{1}{2} \times 90 \times 100$ sq. rd., or 4500 sq. rd., which is equal to $28\frac{1}{8}$ A.

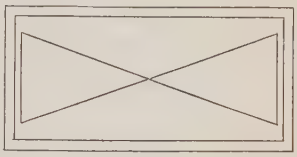
Therefore, to find the area of a field from a drawing,

Draw the plan to scale; divide the plan into triangles; from the base and height of each triangle on the plan compute the base and height of each triangle in the field; from these results find the areas of the several triangles and thus find the area of the field.

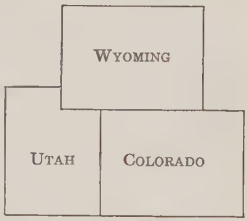
It must be understood that surveyors have better methods, but this method is sufficient for our immediate purposes. The immediate object in view is not to make practical surveyors but to show the general power of mathematics.

Exercise 28. Areas

1. This plan represents a space 150 ft. long and 75 ft. wide, with two triangular flower beds, in a city park. Around the inside of the space is a sidewalk $6\frac{1}{4}$ ft. wide. Measure the figure, determine the scale used in drawing the plan, and find the area of each of the flower beds in the park.



2. This map is drawn to the scale 1 in. = 520 mi. Carefully measure the map and determine approximately the length of each side of each state, and then find the approximate area of each state.

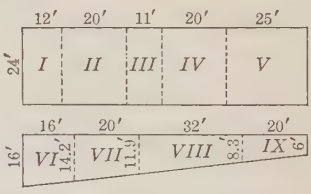


The results obtained will be, of course, merely approximate, since the map is so small. The method is, however, the one which is employed in practical work with larger maps.

3. Each side of a brick building with a slightly sloping roof is in the form of a trapezoid, as here shown. The building is 57 ft. wide, 57 ft. high on the front, and 52 ft. high on the rear. On this side there are 4 windows each 4 ft. wide and 9 ft. high and 4 windows of the same width but $6\frac{1}{2}$ ft. high. If it takes 14 bricks per square foot of outside surface to lay the wall, how many bricks will be needed to lay this wall, deducting for the 8 windows?

4. The sides of a triangular city lot are respectively 72 ft., 60 ft., and 48 ft. Draw a plan of the lot to the scale of 1 in. to 12 ft., measure the altitude of the scale drawing, and find the altitude and area of the lot.

5. This sketch shows the plan of some small suburban garden plots which are offered for sale at 20¢ a square foot. Find the price of each lot.



6. In a certain city Washington Street runs east and west and intersects Third Avenue at right angles. Using the scale 1 in. = 100 ft., draw a plan of the property on the southeast corner from the following description: Beginning at the corner, run south 160 ft., then east 75 ft., then north 15 ft., then east 50 ft., then by a slanting line to a point on Washington Street 100 ft. from the corner, and then to the corner. Find the area of the plot and the value at $2.20 per square foot.

7. In order to measure the distance AB across a swamp some boys measure a line CD, drawn as shown in the figure, and find it to be 280 ft. long. They find that $DA = 40$ ft. and $CB = 90$ ft., DA and CB being perpendicular to CD. Draw the plan to some convenient scale and determine the distance AB. Find also the area of the trapezoid $ABCD$.

8. A swimming tank is 60 ft. long and 35 ft. wide. Draw a plan to the scale 1 in. = 10 ft., determine the length of the diagonal by measurement, and then compute the number of yards that a student will swim in swimming along the diagonal of the tank eight times.

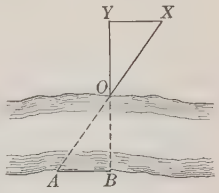
9. A lot has a frontage of 65 ft. and a depth of 150 ft., and a path runs diagonally across it. Draw the plan to scale and find, by measurement, the distance saved by using the path instead of walking round the two sides at a distance of 2 ft. outside the edges of the lot.

10. Suppose that you have 360 ft. of wire screen to inclose a plot in which to keep chickens. If you wish to inclose the largest possible area in the form of a parallelogram, triangle, or trapezoid, which form would you use? Show by a drawing on squared paper that the form which you choose incloses a larger area than the others. Remember that there are several kinds of triangles, several kinds of parallelograms, and several kinds of trapezoids.

11. Draw three different triangles, each with base 2 in. and height 1 in. Find the area of each. What do you infer as to the equality of the areas of triangles having equal bases and equal heights? Write the statement in full.

12. Upon the same base of 2 in. draw three different parallelograms, each having a height $1\frac{1}{2}$ in. Find the area of each parallelogram. What do you infer as to the equality of the areas of parallelograms on the same base and with equal heights? Write the statement in full.

13. For computing the area covered by 1000 ft. of a river, some boys at O wish to find the width OB of the river, as here shown. They know that the distance AB is 300 ft. and that the angle at B is 90°. Show how, by sighting along YB and XA and by making certain measurements, the boys can find the distance OB without crossing the river.



14. Find the area of an equilateral triangle 3.1 in. on a side.

This may be done by drawing the triangle on squared paper and counting the squares, or, more accurately, by first approximating the height by measurement on the squared paper.

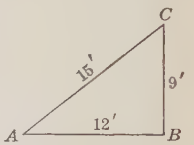
15. Find the area of an isosceles triangle with sides 2 in., 2 in., and $1\frac{1}{2}$ in.

Exercise 29. Optional Outdoor Work

1. Determine the area of your school grounds by carefully making the necessary measurements and dividing the grounds into triangles, if necessary.

2. In Ex. 1 determine the area by drawing the plan to scale.

3. Drive two stakes in the ground at A and B, 12 ft. apart. Fasten one end of a 15-foot line at A and one end of a 9-foot line at B. Draw the loose ends taut and drive a stake where they meet, at C. What kind of an angle is formed at B?



4. Draw the figure of Ex. 3 to the scale which is four times the one here used, and determine from your figure some other measurements which might be used to lay out the same kind of angle. Try this on the school grounds.

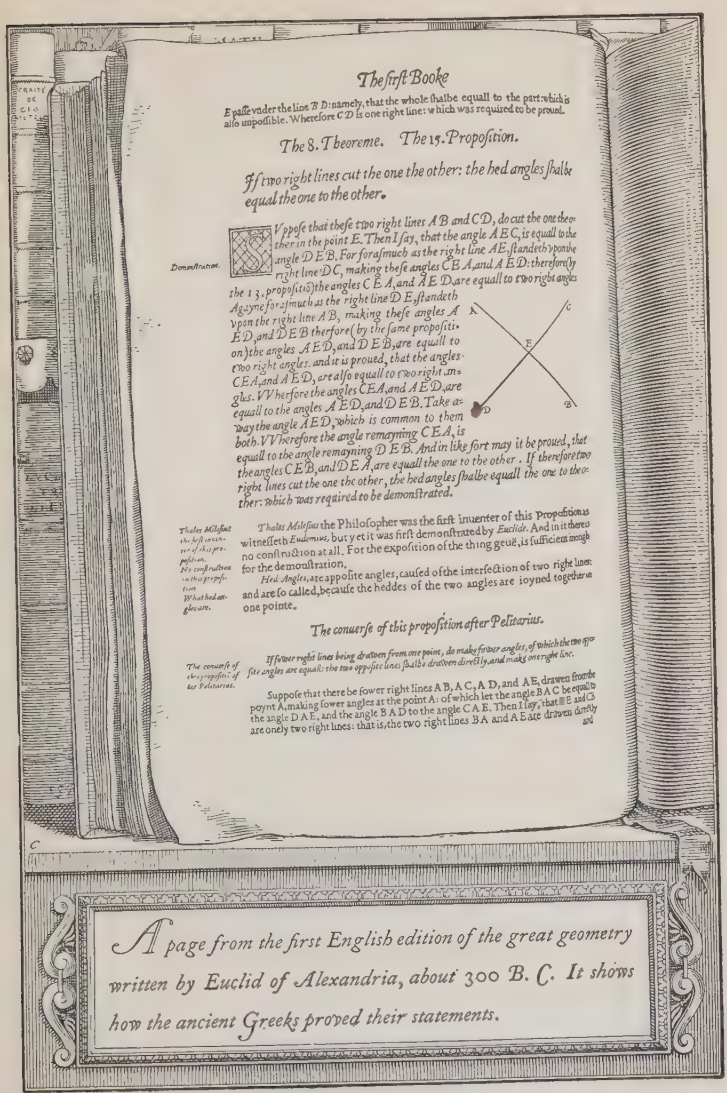
5. What is the largest scale on which a plan of your school grounds could be drawn on a piece of paper 12 in. by 14 in., if you allow for a margin of at least 1 in.?

6. Draw to scale a plan of the lot on which your home stands and indicate the ground plan of the house.

7. Draw to scale a floor plan of some public building in your vicinity. Compute the area covered.

8. Lay off on your school grounds an isosceles triangle, an equilateral triangle, and a right triangle, each with a perimeter of 30 ft. Compute and compare the areas.

9. Lay off on your school grounds several rectangles, each of which has a perimeter of 30 ft. Compute and compare the areas.



The first Booke

E passe vnder the line B D: namely, that the whole shalbe equall to the part: which is also impossible. Wherefore C D is one right line: which was required to be proued.

The 8. Theoreme. The 15. Proposition.

If two right lines cut the one the other: the hed angles shalbe equal the one to the other.

Demonstration.

VPpose that these two right lines A B and C D, do cut the one the other in the point E. Then I say, that the angle A E C, is equall to the angle D E B. For forasmuch as the right line A E, standeth vpon the right line D C, making these angles C E A, and A E D: therefore by the 13. proposition the angles C E A, and A E D, are equall to two right angles. Againe forasmuch as the right line D E, standeth vpon the right line A B, making these angles A E D, and D E B therfore (by the same proposition) the angles A E D, and D E B, are equall to two right angles. and it is proued, that the angles C E A, and A E D, are also equall to two right angles. VVherfore the angles C E A, and A E D, are equall to the angles A E D, and D E B. Take away the angle A E D, which is common to them both. VVherfore the angle remayning C E A, is equall to the angle remayning D E B. And in like sort may it be proued, that the angles C E B, and D E A, are equall the one to the other. If therefore two right lines cut the one the other, the hed angles shalbe equall the one to the other: which was required to be demonstrated.

Thales Milesius the first inuenter of this proposition.

No construction in this proposition.

What hed angles are.

Thales Milesius the Philosopher was the first inuenter of this Propofition as witnesseth *Eudemus*, but yet it was first demonstrated by *Euclide*. And in it there is no construction at all. For the exposition of the thing geuē, is sufficient enough for the demonstration.

Hed Angles, are appolite angles, caused of the interfection of two right lines: and are so called, because the heddes of the two angles are ioyned together in one pointe.

The conuerse of this proposition after Peliterius.

The conuerse of the proposition of Peliterius.

If fower right lines being drawen from one poinct, do make fower angles, of which the two opposite angles are equall: the two opposite lines shalbe drawen directly, and make one right line.

Suppose that there be fower right lines A B, A C, A D, and A E, drawen from the poynt A, making fower angles at the point A: of which let the angle B A C be equall to the angle D A E, and the angle B A D to the angle C A E. Then I say, that ✠ E and C are onely two right lines: that is, the two right lines B A and A E are drawen directly and

A page from the first English edition of the great geometry written by Euclid of Alexandria, about 300 B. C. It shows how the ancient Greeks proved their statements.

10. If the street is to be paved in front of your schoolhouse, what measurements are necessary to determine the area to be covered? Make the measurements for the block in which your schoolhouse stands, draw the plan to scale, and compute the area.

11. If a sidewalk is to be laid in front of your schoolhouse, what measurements are necessary and what prices must be known in order that you may find the cost of the walk? Make the measurements, find by inquiry the prices, and compute the cost of the walk.

Such examples are merely typical of the work which many schools will wish to have done. It is impossible, however, to anticipate the practical cases which may arise in any given locality. They may relate to some building in process of erection, to the laying of a water main in the street, to the reservoir of the city water supply, or to the cost of stone steps for a schoolhouse. The important thing is that the problem should be real and interesting to the class.

12. Compute the number of square feet of the surface of some building which needs to be painted, find the average cost per square yard for painting it one coat, and then compute the cost of painting the building.

13. Suppose that a water main is to be laid in the street in front of the schoolhouse. Ascertain by inquiry the usual width of a trench for such a purpose, and draw a plan of the street to scale, showing the location of the trench and giving it the proper width to scale on the drawing.

14. In the upper picture on the opposite page can you see how the height of the tower could be measured by simply tipping the quadrant over flat and making certain measurements on the ground? Try this plan in measuring the height of some tree or building.

In this case also it may be noticed that the angle is exactly 45°, and so there is another and better way of finding the height.

Illustrations from old books on geometry, showing how the height of a tower or the distance to an island can be found by the aid of a simple instrument which can easily be made.

Ratio. We often hear of the ratio of one number to another, as when some one speaks of the ratio of the width of a tennis court to its length, or the ratio of daylight to darkness in the winter, or the ratio of a man's expenses to his income. By the ratio of 3 to 4 we mean $3 \div 4$, or $\frac{3}{4}$, while the ratio of 1 in. to 1 ft. is $\frac{1}{12}$ and the ratio of $\frac{1}{2}$ to $\frac{3}{4}$ is $\frac{1}{2} \div \frac{3}{4}$, or $\frac{2}{3}$.

The relation of one number to another of the same kind, as expressed by the division of the first number by the second, is called the *ratio* of the first to the second.

A few examples of ratio should be given on the blackboard. Thus the ratio of \$3 to \$6 is $\frac{3}{6}$, or, in its simplest form, $\frac{1}{2}$; the ratio of 1 yd. to 1 ft. is the same as the ratio of 3 ft. to 1 ft., or 3; the ratio of 5 to 2 is $\frac{5}{2}$, or $2\frac{1}{2}$; and the ratio of any number to itself is 1.

The ratio of 2 to 3 may be written in the fraction forms, $\frac{2}{3}$ or 2/3, or it may be written with a colon between the numbers; that is, as 2 : 3.

The teacher should explain to the class that the ratio of 12 ft. to 4 ft., for example, may be written $\frac{12 \text{ ft.}}{4 \text{ ft.}}$, $\frac{12}{4}$, 12 : 4, or simply 3. The word "ratio" is used for each of these forms. The expression 12 : 4 is read "the ratio of 12 to 4," or "as 12 is to 4," 12 and 4 being called the *terms* of the ratio.

Since any number divided by a number of the same kind, as inches by inches or dollars by dollars, has an abstract quotient, we see that

A ratio is always abstract, and its terms may therefore be written as abstract numbers.

That is, instead of labeling our numbers, as in 2 ft. : 4 ft., we may omit all labels and write simply 2 : 4, or $\frac{2}{4}$, or $\frac{1}{2}$.

Teachers should use the familiar fraction form first. Indeed, the special symbol (:) is slowly going out of use because it is not necessary. We often see 2 : 3 written as 2/3 instead of $\frac{2}{3}$. Ratios are little more than fractions and may be treated accordingly.

Exercise 30. Ratio

All work oral

1. Expressed in simplest form, what is the ratio of 6 to 12? of 12 to 6?

When a ratio is asked for, the result should always be stated in the simplest form unless the contrary is expressly stated.

2. What is the ratio of \$4 to \$12? of 4 ft. to 12 ft.?

3. What is the ratio of $4\frac{1}{2}$ to 9? of 15 to $7\frac{1}{2}$?

4. In the figure below, what is the ratio of E to D? What is the ratio of E to C?

When we speak of the ratio of E to D we mean the ratio of their number values; that is, of 1 to 2, the ratio being $\frac{1}{2}$. When we speak of the ratio of E to $2\,B$ we mean the ratio of 1 to 2×4. This is $\frac{1}{8}$.

5. In the figure below, what is the ratio of E to B? What is the ratio of E to A? of E to $D + C$?

Referring to the figure, state the following ratios:

6. E to $\frac{1}{2}B$.

7. D to $2\,A$.

8. $2\,E$ to D.

9. $2\,D$ to B.

10. C to $3\,E$.

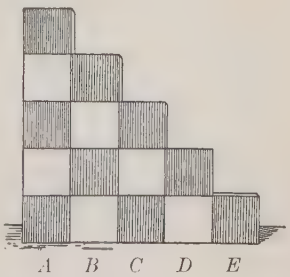
11. A to E.

12. C to B.

13. D to C.

14. C to A.

15. A to $2\,D$.



16. What is the ratio of any number to twice itself?

17. What is the ratio of a foot to a yard? of an inch to a foot? of 8 oz. to 1 lb.? of 1 pt. to 1 qt.? of 2 qt. to 1 gal.?

In every such case the measures must be expressed in the same units before the ratio is found. Thus the ratio of 1 yd. to 7 ft. is the ratio of 3 ft. to 7 ft., or of 1 yd. to $2\frac{1}{3}$ yd., either of which is $\frac{3}{7}$.

Proportion. An expression of equality between two ratios is called a *proportion.*

For example, $\$5 : \$8 = 10$ ft. : 16 ft. is a proportion. This proportion is read "$\$5$ is to $\$8$ as 10 ft. is to 16 ft." or "the ratio of $\$5$ to $\$8$ is equal to the ratio of 10 ft. to 16 ft." It may, of course, be written simply $5 : 8 = 10 : 16$, or $\frac{5}{8} = \frac{10}{16}$.

The first and last terms of a proportion are called the *extremes*; the second and third terms are called the *means*. These expressions are unnecessary, however, in the treatment of the subject in the junior high school.

We often have three terms of a proportion given and wish to find the fourth. For example, we may have the proportion

$$n : 14 = 27 : 63,$$

where n represents some number whose value we wish to find. We may write the proportion in the more familiar fraction form, thus:

$$\frac{n}{14} = \frac{27}{63}.$$

If, now, $\frac{1}{14}$ of n is equal to $\frac{27}{63}$, we see that n must be equal to $14 \times \frac{27}{63}$, or 6.

The teacher should show on the blackboard that we need merely multiply the two equal ratios by 14, canceling as much as possible, and we have $n = 6$.

If we have $4 : n = 12 : 6$, we may simply take the ratios the other way, and have $n : 4 = 6 : 12$, and then solve as above.

The old method of solving business problems by ratio and proportion is no longer used to any considerable extent. The subject of ratio has a value of its own, however, and proportion is peculiarly useful in geometry.

It is interesting to notice that *in any proportion of abstract numbers the product of the first and fourth terms is equal to the product of the second and third terms.*

Exercise 31. Proportion

Find the value of n in each of the following proportions:

1. $n:18 = 7:9$.
2. $n:42 = 13:14$.
3. $7:n = 9:72$.
4. $15:13 = n:65$.

5. A certain room is 24 ft. by 32 ft. and the width is represented on a drawing by a line 9 in. long. How long a line should represent the length?

6. When a tree 38 ft. high casts a shadow 14 ft. long, how long is the shadow cast by a tree 64 ft. high?

In all such cases the trees are supposed to be in the same locality and perpendicular to a level piece of ground.

7. If a picture 42 in. by 96 in. is reduced photographically so that the length is $7\frac{1}{2}$ in., what is the width?

8. By means of a pantograph a student enlarges the floor plans for a house in the ratio of 8 : 3. If the dining room in the original plans measures $2\frac{1}{2}$ in. by 3 in., what are the dimensions in the enlarged drawing?

9. The sides of a triangle are 9 in., 7 in., and 6 in. Construct a triangle the corresponding sides of which are to the sides of the given triangle as 3 : 4.

10. A map is drawn to the scale of 1 in. to 0.8 mi. How many acres of land are represented by a portion of the map 1 in. square?

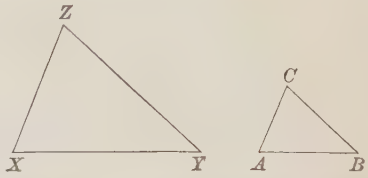
1 mi. = 320 rd., and 1 A. = 160 sq. rd.

11. The floor of a schoolroom is 24 ft. by 30 ft. The total window area is to the floor area as 1 : 5, and the 6 windows have equal areas, each window being $3\frac{1}{2}$ ft. wide. Determine the height of each window to the nearest quarter of an inch.

Proportional Numbers. Numbers which form a proportion are called *proportional numbers.*

Similar Figures. As stated on page 141, figures which have the same shape are called *similar figures* and are said to be *similar.*

For example, these two triangles are similar. Likewise triangles ABC and XYZ on page 187 are similar.



Proportional Lines. The lengths of corresponding lines in similar figures are proportional numbers; that is, *corresponding lines in similar figures are proportional.*

For example, in the above triangles $XY : YZ = AB : BC$. In two circles the circumferences and radii are proportional, the circumference of the first being to the circumference of the second as the radius of the first is to the radius of the second.

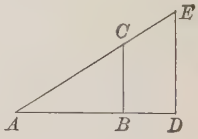
Exercise 32. Similar Figures

Examples 1 to 4, oral

1. In the above triangles, if XY is twice as long as AB, how does ZX compare in length with CA?

2. In the figure below state two proportions that exist among AB, AD, AC, and AE.

3. In this figure, if AB is $\frac{2}{3}$ of AD, what is the ratio of AC to AE?



4. In the same figure, if DE represents the height of a man 6 ft. tall, BC the height of a boy, DA the length of the shadow cast by the man, and BA the length of the shadow cast by the boy, show how to find the height of the boy by measuring the lengths of the shadows.

5. If a tree BC casts a shadow 35 ft. long at the same time that a post YZ which is 12 ft. high casts a shadow 15 ft. long, how high is the tree?

Suppose YZ to be the post, XY to be its shadow, and AB to be the shadow of the tree.

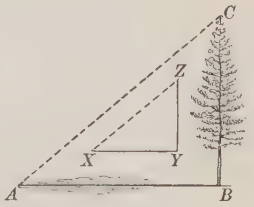
Since the triangles ABC and XYZ are similar, we may find h, the height of the tree, from the proportion

$$\frac{BC}{AB} = \frac{YZ}{XY},$$

or by writing the values,

$$\frac{h}{35} = \frac{12}{15};$$

whence $$h = \frac{\overset{7}{\cancel{35}} \times \overset{4}{\cancel{12}}}{\underset{3}{\cancel{15}}} = 28.$$



That is, the tree is 28 ft. high.

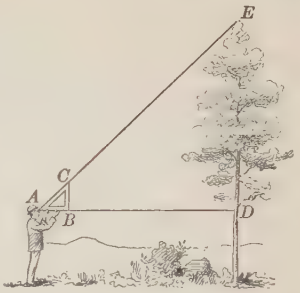
6. If a tree casts a shadow 58 ft. long at the same time that a post 8 ft. high casts a shadow 14 ft. 6 in. long, how high is the tree? Draw the figure to scale.

7. If a telephone pole casts a shadow 27 ft. long at the same time that a boy 5 ft. tall casts a shadow 4 ft. 6 in. long, how high is the pole? Draw the figure to scale.

8. A boy threw a ball directly upward and watched its shadow on the sidewalk. When the ball began to descend, the shadow of the ball was at a fence post 32 ft. away. The boy was 4 ft. 6 in. tall and his shadow was 2 ft. 3 in. long. How high did the boy throw the ball above the level of the ground? Draw the figure to scale.

9. A water tower casts a shadow 87 ft. 8 in. long at the same time that a baseball bat placed vertically upright casts a shadow twice its own length on a level sidewalk. Find the height of the water tower. Draw the figure to scale.

10. This man is holding a right triangle ABC in which $AB = BC$. What is the height of the tree in the picture if the base of the triangle is 5 ft. 3 in. from the ground and if AD is 32 ft.?



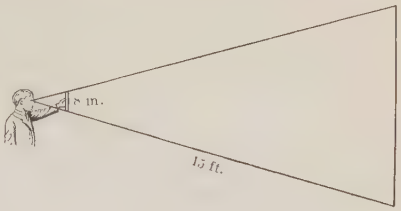
This is a common way employed by woodsmen for measuring the heights of trees. The man backs away from the tree until, holding the triangle ABC so that AB is level, he can just see the top of the tree along the side AC.

In all problems involving heights and distances the student should estimate the result in advance. This will serve as a check on the accuracy of the work.

11. In Ex. 10 suppose that a triangle is used which has AB equal to twice BC, that AD is 62 ft., and that the point B is 5 ft. 7 in. above the ground; find the height of the tree.

12. A woodsman wishes to determine the distance from the ground to the lowest branch of a tree. He finds that if he places a stick vertically in the ground at a distance of 32 ft. from the tree, lies on his back with his feet against the stick, and sights over the top of the stick, the line of sight will strike the tree at the lowest limb, as shown in the figure. The woodsman's eye is 5 in. above the ground, the distance EF, as shown in the figure, is 5 ft. 6 in., and the top of the stick is 4 ft. 9 in. above the ground. Determine the distance BC from the ground to the lowest branch.

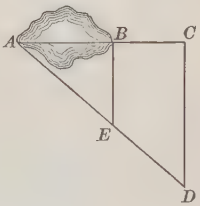
13. A boy whose eye is 15 ft. from the bottom of a wall sights across the top and bottom of a stick 8 in. long and just sees the top and bottom of the wall, the stick being held parallel to the wall as shown. If the bottom of the stick is 18 in. from the eye, what is the height of the wall?



14. A woodsman steps off a distance of 30 ft. from a tree, faces the tree, and holds his ax handle at arm's length in front of him parallel to the tree. His hand is 27 in. from his eye, and 2 ft. 4 in. of the ax handle just covers the distance from the ground to the lowest limb of the tree. How high is the lowest limb of the tree?

This method suffices for a fair approximation to the height.

15. Wishing to find the length AB of a pond, some boys choose a point C in line with A and B, and at B and C draw lines perpendicular to BC, and draw AD. By measuring they then find BC to be 84 ft., DE to be 112 ft., and EA to be 154 ft. What is the length of the pond?



16. In Ex. 15 what other measurements may be used to find the distance AB?

17. If $\frac{3}{4}$ in. on a map represents a distance of 375 mi., how many miles will $2\frac{1}{2}$ in. represent?

18. If a tree casts a shadow 40 ft. long when a post 5 ft. high casts a shadow $6\frac{1}{4}$ ft. long, how high is the tree?

19. If $1\frac{5}{8}$ in. on a map represents a distance of 325 mi., how many inches represent a distance of 340 mi.?

20. In one of the upper illustrations on the opposite page suppose the length of the shadow of the post to be 1 ft. 6 in. shorter than the height of the post, and suppose the shadow of the tower to be 69 ft. 4 in. and the height of the post to be 5 ft. 2 in. Find the height of the tower.

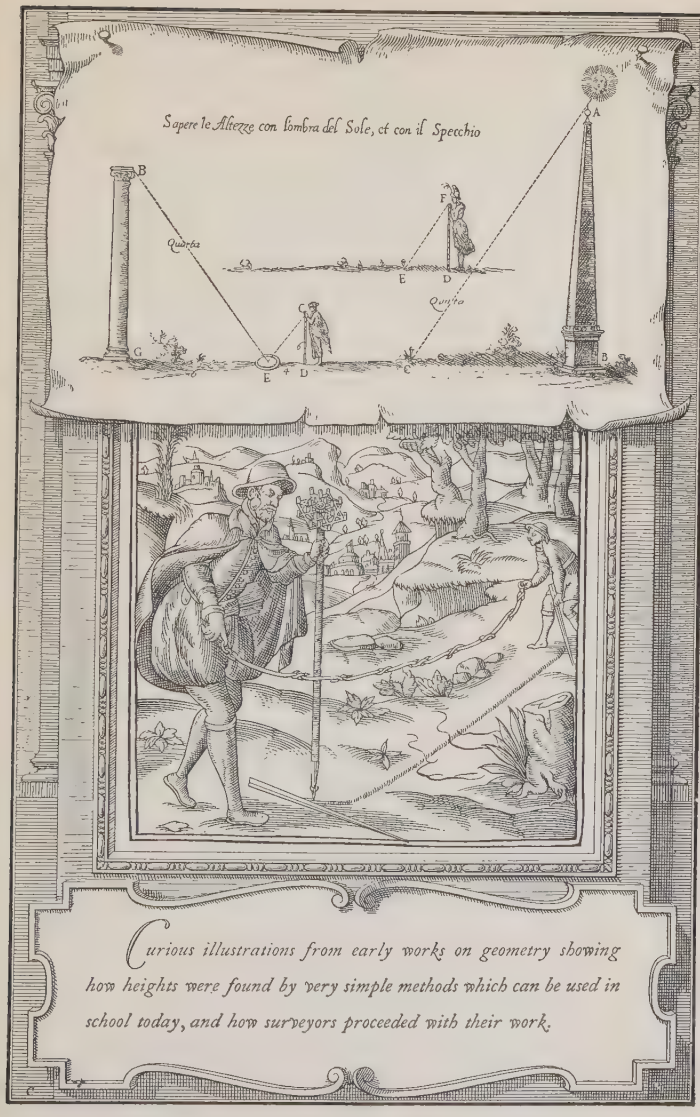
21. In one of the upper illustrations on the opposite page there is also shown a very old method of finding the height by means of a mirror placed level on the ground. Can you see two similar triangles in the picture? If so, describe the method by which you could find the height of a tree in this way.

22. Some members of a class made a right triangle with one side 9 in. and the other side 12 in. One of them held the triangle so that the longer side was vertical and then backed away from a tree until he could just see the top by sighting along the hypotenuse. The class then measured and found that the eye of the observer was 45 ft. in a horizontal line from the tree and 4 ft. 10 in. from the ground. Draw the figure to scale and find the height of the tree.

23. Draw a plan of the top of your desk to scale, representing the length by 3 in. What will be the width of the drawing, and how can it be found?

24. The extreme length of a new leaf is 2 in. and the extreme width is 1 in. After the leaf has grown $\frac{1}{2}$ in. longer, maintaining the same shape, what is its width?

25. A girl is making an enlarged drawing from a photograph of a friend. In the photograph the distance between the eyes is $\frac{2}{3}\frac{1}{2}$ in. and the length of the nose is $\frac{7}{16}$ in. If the distance between the eyes in the drawing is 50% more than it is in the original, what is the length of the nose in the drawing?



Curious illustrations from early works on geometry showing how heights were found by very simple methods which can be used in school today, and how surveyors proceeded with their work.

Exercise 33. Optional Outdoor Work

1. Measure the height of any tree, telegraph pole, or church spire in the vicinity of the school building, using any convenient method.

In such cases it is desirable to have the class discuss the methods in the class hour preceding the outdoor work, deciding upon the methods to be used. It is then a good plan to separate the class into groups, each group using a different method from the others. The results can then be compared and, if the methods and work are equally good, an average may be taken as a fair approximation.

2. Measure the distance from one point in the vicinity of the school, preferably on the school grounds, to another point so situated that a line cannot be run directly between them. Use any convenient method.

In case no such points can be found, the distance across the street may be measured without actually crossing.

3. Making the necessary measurements, find the area of the school grounds or the area of such a portion as is decided upon by the teacher and the class.

In suburban or rural communities the areas of fields may be found. The class should see that it now has mastered enough mathematics for finding the area of any ordinary field.

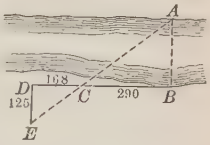
4. Ascertain the cost of a concrete sidewalk, per square foot or square yard, and compute the cost of a good sidewalk in front of the school.

In case any excavations are being made for buildings near the school, and it is feasible to have the class make the necessary measurements, the amount of earth removed may be computed.

In some schools this optional work may be practicable, while in others it may not be. Teachers will have to be guided by circumstances in assigning the above and similar exercises.

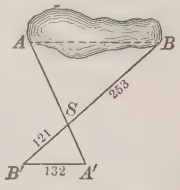
The problems on page 193 are typical of those which may be considered for outdoor work.

5. To find the distance across a river measured from B to A, a point C was so chosen by the class that BC was perpendicular to AB at B. Then a perpendicular to BC prolonged was drawn. The class sighted from C to A and placed a stake at E where the line of sight from C to A cut the perpendicular from D. By measurement DC was then found to be 168 ft. and CB 290 ft. and DE 125 ft. What was the distance across the river, measured from B to A?



6. In order to determine the distance from A to B on opposite sides of a hill, what measurements indicated in this figure are necessary? Make a problem involving this principle, with reference to two points near the school, take the necessary measurements out of doors, solve the problem, and check the work by measuring the figure drawn to scale.

7. Wishing to determine the length AB of a pond, a class placed a stake at S, as shown in the figure. The line BS was then run on to B', 121 ft. from S, and BS was measured and found to be 253 ft. By sighting from B, the angle B was marked off on a piece of cardboard, and then the angle B' was made equal to it, $B'A'$ being thus drawn to a point A' exactly in line with A and S. By measurement $B'A'$ was found to be 132 ft. Find the length of AB.

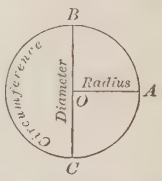


This should be considered at the blackboard before solving. Notice the advantage of using A' to correspond to A, and B' to B.

8. Make a problem similar to Ex. 7, take the necessary measurements, and solve.

Circumference, Diameter, and Radius. The line bounding a circle is called the *circumference*. Any line drawn through the center of a circle and terminated at each end by the circumference is called a *diameter*, and any line drawn from the center to the circumference is called a *radius*.

Ratio of Circumference to Diameter. Cut from cardboard several circles with different diameters. Mark a point P on each circumference, roll the circle along a straight line, and determine the length of the circumference by measuring the line between the points where P touches it. In each case the circumference will be found to be approximately $3\frac{1}{7}$ times the diameter.



The number 3.1416 is a closer approximation, but $3\frac{1}{7}$ should be used unless otherwise stated.

Since all circles have the same shape,

$$\frac{\text{Any circumference}}{\text{Diameter of its circle}} = \frac{\text{Any other circumference}}{\text{Diameter of its circle}} = 3\tfrac{1}{7}.$$

A special name is given to this ratio $3\frac{1}{7}$; it is called pī (written π), a Greek letter. That is, $c : d = \pi$,

or $$c = \pi d.$$

Since the diameter is twice the radius, $d = 2r$; therefore

$$c = 2\pi r.$$

1. Find the circumference of a bicycle wheel the diameter of which is 28 in.

We have $c = \pi d = \frac{22}{7} \times 28 \text{ in.} = 22 \times 4 \text{ in.} = 88 \text{ in.}$

2. Find the radius which was used in constructing the base of a circular water tank 24.2 ft. in circumference.

Since $c = 2\pi r$, it follows that $r = c \div 2\pi$. Therefore we have

$$r = 24.2 \text{ ft.} \div (2 \times \tfrac{22}{7}) = 3.85 \text{ ft.}$$

Exercise 34. Circles

Examples 1 to 8, oral

State the circumferences of circles of the following diameters:

1. 7 in. **2.** 21 in. **3.** 42 in. **4.** 56 in.

State the circumferences of circles of the following radii:

5. 14 in. **6.** 7 in. **7.** 21 in. **8.** 28 in.

Find the circumferences, given the following diameters:

9. 68.6 in. **11.** 9.38 ft. **13.** 53.9 ft. **15.** 128.8 ft.
10. 420 in. **12.** 3.01 ft. **14.** 13 ft. 5 in. **16.** 116.2 ft.

Find the diameters, given the following circumferences:

17. 176 in. **18.** 770 yd. **19.** 3.96 ft. **20.** 48.4 ft.

Find the circumferences, given the following radii:

21. 77 in. **22.** 105 in. **23.** 1.75 in. **24.** 126 in.

25. What is the circumference of a 56-inch wheel?

26. Given that the inner circumference of a circular running track is half a mile, find the diameter.

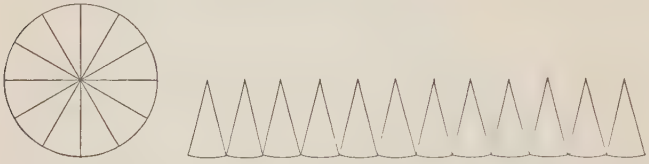
27. A girl has a bicycle with 26-inch wheels. How many revolutions will each wheel make in going a mile?

28. A circular pond 30 ft. in diameter is surrounded by a path 6 ft. wide. What is the length of the outer circumference of the path?

29. How many revolutions will an automobile wheel 37 in. in diameter make in going one mile?

30. If the cylinder of a steam roller is 6.5 ft. in diameter and 8 ft. long, how many square feet of ground will it roll at each complete revolution?

Area of a Circle. A circle can be separated into figures which are nearly triangles. The height of each triangle is the radius, and the sum of the bases is the circumference. If these figures were exact triangles the area of the circle would



be $\frac{1}{2} \times$ height $\times$ sum of bases; and, since they are nearly triangles whose bases together are the circumference, we may say that the area is $\frac{1}{2} \times$ radius $\times$ circumference. It is proved later in geometry that this is the true area.

We may now express this by a formula, thus:

$$A = \tfrac{1}{2} rc.$$

Since $c = 2\pi r$ we may put $2\pi r$ in place of c, and have

$$A = \tfrac{1}{2} r \times 2\pi r = \pi r^2.$$

The teacher should explain to the class, if necessary, that the *square* of a number is the product obtained by multiplying the number by itself, and that it is customary to write 3^2 for the square of 3.

1. A tinsmith in making the bottom of a tin pail draws the circle with a radius of 5 in. How much tin does he need, not counting waste?

Since $A = \pi r^2$,

we have $A = \frac{22}{7} \times 5 \times 5$ sq. in. $= 78\frac{4}{7}$ sq. in.

2. In order to have an iron pillar capable of supporting a certain weight, the cross section must be $50\frac{2}{7}$ sq. in. What radius should be used in drawing the pattern?

Since $A = \pi r^2$, we have $A \div \pi = r^2$, or $50\frac{2}{7} \div \frac{22}{7} = r^2$, and so $16 = r^2$. Because $16 = 4 \times 4$, we see that $r = 4$.

Exercise 35. Area of a Circle

Examples 1 and 2, oral

State the areas of circles, given the radii as follows:

1. 7 in. **2.** 1 in. **3.** 1.4 in. **4.** 2.8 in. **5.** 56 in.

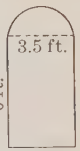
6. If the radius of one circle is twice as long as the radius of another circle, how do the areas of the circles compare? How do the circumferences compare?

7. If one circle has a radius three times as long as the radius of another circle, how do the areas compare? How do the circumferences compare?

8. A circular mirror is 2 ft. 3 in. in diameter. Find the cost of resilvering the mirror at 36¢ a square foot.

9. What is the area of the cross section of a 3-inch water pipe?

10. Find the entire area of a window the lower part of which is a rectangle 3.5 ft. wide and 6 ft. high, and the upper part a semicircle, or half circle, as shown in the figure.



11. In a park there is a circular lake of diameter 120 ft. Find the number of square yards of surface in a walk 5 ft. wide around the lake.

12. In the lake of Ex. 11 there is a circular island with radius 25 ft. What is the surface area of the water?

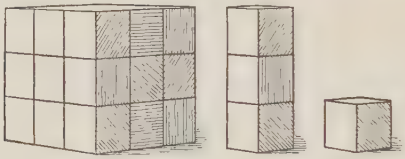
13. What is the cost, at $2.10 a yard, of erecting a wall around the lake of Ex. 11?

14. A circular tree has a circumference of 12 ft. 3 in. at a certain height above the ground. What will be the area of the top of the stump made by sawing horizontally through the tree at this point?

Exercise 36. Volumes

All work oral

1. If $C = 1$ cu. in., what is the volume of B? of A?



A *B* *C*

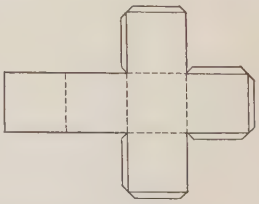
2. Find the volume of a workbox 3 in. by 4 in. by 2 in.

State the volumes of solids of the following dimensions:

3. 4 in., 5 in., 6 in.
4. 3 in., 3 in., 7 in.
5. 2 in., 3 in., 10 in.
6. 6 in., 8 in., 10 in.

Rectangular Solid. A solid having six sides, each side being a rectangle, is called a *rectangular solid*. If all the sides are squares the rectangular solid is called a *cube*.

A cube may be constructed from cardboard by cutting out a diagram like this, bending the cardboard on the dotted lines, and pasting the flaps. In a similar way any rectangular solid may be constructed.



Volume of a Rectangular Solid. From the exercise given above we see that

The volume of a rectangular solid is equal to the product of the three dimensions.

Expressed as a formula, using initial letters, we have

$$V = lbt.$$

Exercise 37. Volumes

Examples 1 to 9, oral

State the volumes of solids of the following dimensions:

1. 3″, 4″, 6″.	**4.** 7′, 2′, 3′.	**7.** 3 yd., 7 yd., 2 yd.
2. 10″, 4″, 5″.	**5.** 20′, 30′, 4′.	**8.** 4 in., 5 in., 8 in.
3. 3″, 8″, 10″.	**6.** 9″, 3″, 2″.	**9.** 9 ft., 4 ft., 3 ft.

10. A cellar 24 ft. by 32 ft. by 6 ft. is to be excavated. How much will the excavation cost at 45¢ a load?

Consider a load as equal to 1 cu. yd.

11. How much will it cost, at 52¢ a cubic yard, to dig a ditch 180 ft. long, 3 ft. wide, and 5 ft. deep?

12. The box of an ordinary farm wagon is 3 ft. by 10 ft. and the depth is usually 24 in. or 26 in. Find the contents in cubic feet and cubic inches for each of these depths.

13. The interior of a certain freight car is 36 ft. long, 8 ft. 4 in. wide, and 7 ft. 6 in. high. How many cubic feet does it contain? If it is filled with grain to a height of $4\frac{1}{2}$ ft., what is the weight of the grain at 60 lb. to the bushel, allowing $1\frac{1}{4}$ cu. ft. to the bushel?

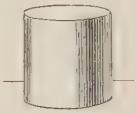
14. To what depth must a tank 5 ft. wide and 6 ft. 8 in. long be filled with water to contain 120 cu. ft. of water?

15. It is estimated that $2\frac{2}{5}$ cu. ft. of corn in the ear will produce 1 bu. of shelled corn. How many bushels of shelled corn can be obtained from a crib 10 ft. by 18 ft. by 7 ft., filled with corn in the ear?

16. A cellar 22 ft. by 30 ft. by 7 ft. is to be dug for a house on a level lot 62 ft. by 140 ft. The dirt taken from the cellar is to be used on the rest of the lot. To what depth will the lot be filled if the dirt is evenly distributed?

Cylinder. A solid which is bounded by two equal parallel circular faces and a curved surface is called a *cylinder.*

Only the right cylinder is considered in this book.



The two parallel circular faces are called the *bases*, and the distance between the bases is called the *height* or *altitude* of the cylinder.

Volume of a Cylinder. Since we can place 1 cu. in. on each square inch of the lower base if the cylinder is 1 in. high, we see that if the cylinder is 5 in. high we can place 5 times as many cubic inches. Hence we see that

The volume of a cylinder is equal to the area of the base multiplied by the altitude.

Expressed as a formula, using initial letters, we have

$$V = bh.$$

Since the base b is a circle, it is equal to πr^2, and so

$$V = \pi r^2 h.$$

Exercise 38. Volume of a Cylinder

1. What is the capacity of a cylindric gas tank 60 ft. in diameter and 38 ft. 6 in. high?

In all such cases the inside dimensions are given.

2. Find the capacity in gallons (231 cu. in.) of a water tank of diameter 24 ft. 3 in. and height 66 ft. 9 in.

3. A water pipe 16 ft. long has a diameter of 1 ft. 9 in. How many cubic inches of water will it contain?

4. A farmer built a silo 24 ft. high and 14 ft. in diameter. If 1 cu. ft. of silage averages 37 lb., how long will the silage that fills the silo last 52 cows at 38 lb. per cow per day?

Such problems should be omitted in places where the students are not familiar with silos.

Curved Surface of a Cylinder. If we cut a piece of paper so that it will just cover the curved surface of a cylinder, how may we find the area of the paper? What are the dimensions of a piece of paper that will just cover the curved surface of a cylinder 6 in. high and having a circumference of 8 in.? What is the area of the paper? What is the area of the curved surface of the cylinder?

We see that *the area of the curved surface of a cylinder is equal to the circumference multiplied by the height.*

That is, $\text{area} = \text{circumference} \times \text{height},$

or $\text{area} = \frac{22}{7} \times \text{diameter} \times \text{height}.$

Expressed as formulas the above statements become

$$A = ch, \quad A = \pi dh, \quad \text{or} \quad A = 2\pi rh.$$

Exercise 39. Curved Surface of a Cylinder

All work oral

1. How many square feet in the curved surface of a wire 1 in. in circumference and 600 ft. in length?

2. How many square feet of tin in a pipe of length 12 ft. and circumference 6 in., allowing 1 sq. ft. for the seam?

3. A tin cup is 4 in. high and 7 in. around, including allowance for soldering. How many square inches of tin are needed for the curved surface?

4. A tin water pipe has a circumference of 9 in. and a length of 10 ft., both measures including allowance for soldering. How many square inches of tin were used?

State the areas of the curved surfaces of cylinders with heights and circumferences as follows:

5. 36 in., 80 in. 6. 40 in., 50 in. 7. 40 in., 35 in.

Exercise 40. Volume and Surface of a Cylinder

1. How many square feet of sheet iron will be required to make a stovepipe $4\frac{1}{2}$ ft. long and 7 in. in diameter, if we allow $\frac{1}{2}$ in. for the lap in making the seam?

2. A certain room in a factory is heated by 246 ft. of steam pipe of diameter 2 in. Find the radiating surface; that is, the area of the curved surface which radiates heat.

3. Compare the surface of a cylinder 12 ft. long, having a radius of 20 in., and the combined surfaces of 10 cylinders each 12 ft. long and each having a radius of 2 in.

4. A farmer has a solid concrete roller 2 ft. 2 in. in diameter and 6 ft. wide. How many cubic feet of concrete are there in the roller? How many square yards of land does it roll in going $\frac{1}{2}$ mi.?

5. At 40¢ a square yard, find the cost of painting a cylindric standpipe 64 ft. high and 9 ft. 3 in. in diameter.

6. State a rule and a formula for finding the circumference of a cylinder, given the radius, and also a rule and a formula for finding the area of the circular cross section.

7. State a rule and a formula for finding the total surface of a cylinder, given the radius of the base and the height.

8. State a rule and a formula for finding the volume of a cylinder, given the diameter and the height?

9. If you know the volume of a cylinder and the circumference of the base, how do you find the height?

10. A large suspension bridge has 4 cables, each 1942 ft. long and 1 ft. 3 in. in diameter. In letting the contract for painting these cables, it is necessary to know their surface. Compute it.

Lathing and Plastering. Laths are usually 4 ft. long, $1\frac{1}{2}$ in. wide, and $\frac{1}{4}$ in. thick and are sold in bunches of 100. Since the laths are laid $\frac{1}{2}$ in. apart, 1 lath is required for a space 2 ft. long and 4 in. wide.

Metal lathing is also commonly used.

Plastering is commonly measured by the square yard, and there is no uniform practice in regard to the method of making allowance for openings. The allowance to be made should be mentioned in the contract.

Exercise 41. Lathing and Plastering

1. How many laths are required to cover a space 16 ft. long and 10 ft. wide?

2. How many laths are required for the walls and ceilings of the living room, dining room, and kitchen shown in the plan on page 204 if the rooms are 9 ft. high and no allowance is made for openings, baseboards, or waste?

3. At 50¢ per square yard, compute the cost of plastering the walls and ceilings of the reception hall, living room, and dining room shown on page 204, no allowance being made for openings or baseboards.

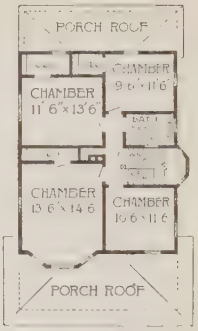
4. At 30¢ per square yard, compute the cost of rough plastering the kitchen shown on page 204, allowing for the four doors, which are 7 ft. high and 3 ft. wide and for the two windows, which are 6 ft. high and 3 ft. wide.

5. Find the total cost of the materials required to lath and plaster a room at the following prices: 20 bu. of lime at 45¢ per bushel; $3\frac{1}{4}$ cu. yd. of sand at 65¢ per cubic yard; 4 bu. of hair at 45¢ per bushel; 200 lb. of plaster of Paris at 55¢ per hundred; 2800 laths at $2.90 per thousand; 20 lb. of nails at 16¢ per pound.

Exercise 42. Practical Building

1. Determine the scale to which the architect drew the plans for a two-story frame house which are shown below.

FIRST FLOOR PLAN



SECOND FLOOR PLAN

2. Compute the cost, at 40¢ a cubic yard, of excavating, 1 ft. beyond the walls, for a 7-foot cellar under the main part of the house, but not under the pantry and porches.

3. Compute the cost, at 62¢ a square yard, of cementing the floor of the cellar if the cellar walls are 1 ft. thick.

4. Compute the cost of flooring with hard wood $\frac{7}{8}$ in. thick the reception hall, living room, and dining room at \$90 per M board feet, allowing 25% for waste.

A board foot (bd. ft.) is the measure of a piece of lumber 1 ft. long, 1 ft. wide, and 1 in. or less thick. The number of board feet in a board less than 1 in. thick is the same as in a board of the same length and breadth but 1 in. thick. A fraction of a board foot is counted a whole board foot.

5. Draw a plan of the first floor on a scale three times as large as the one above, using a pantograph if desired.

6. Draw a plan of the second floor on a scale six times as large as the one above, using a pantograph if desired.

Metric Measures. We are familiar with our common measures of length, including inches, feet, yards, and miles; of weight, including ounces, pounds, and tons: of capacity, including quarts, gallons, and bushels: of surface, including square inches, square yards, and acres. We need to know something, however, about the measures used in countries with which we have extensive business relations. During the European war the newspapers spoke often of the 75-millimeter guns, the 305-millimeter guns, the 700-kilogram shells, and the capture of 500 meters of trenches. None of this means much to us unless we know what millimeters, kilograms, and meters represent in our measures.

Our cities carry on a great deal of business with foreign countries, and since those countries buy our goods, we must be able to describe them in terms of foreign measures, especially because of the recent remarkable increase in our foreign trade.

The work on pages 205–211 is optional. It is not required in subsequent work in this book, but it should be taken if time permits.

Metric Measures of Length. A *meter* (m.) is equal to 39.37 in., or about 1 yd. 3 in. One thousandth of a meter is called a *millimeter* (mm.); one hundredth of a meter, a *centimeter* (cm.); one thousand meters, a *kilometer* (km.).

The meter and centimeter should be drawn on the blackboard.

The meter stick should be used in actual measurement in the schoolroom. Students should be told that 1 km. is about $\frac{5}{8}$ mi.

Metric Measures of Weight. A *gram* (g.) is equal to 15.43 grains, and a *kilogram* (kg.) is equal to 1000 g., or 2.2 lb.

Metric Measures of Capacity. A *liter* (l.) is nearly the same as a quart. The prefix *milli* means thousandth, *centi* means hundredth, and *kilo* means thousand, and so we know what a milliliter, centiliter, and kiloliter mean.

A liter is equal to 0.91 of a dry quart or 1.06 liquid quarts, but such equivalents need not be taught to students at this time.

Exercise 43. Approximations

All work oral

1. The newspaper says that a 75-millimeter gun was used by an army. About what was the diameter in inches?

1 m. = 39.37 in., and 75 mm. = 0.075 of 39.37 in., or about $\frac{3}{40}$ of 40 in., or about how many inches?

2. A gun on a battleship has a bore of 300 mm. About what is this in inches?

300 mm. = 0.300 m. = 0.3 m. = about 0.3 of 40 in., or about how many inches?

3. A manufacturer shipped a lot of flatirons weighing 3 kg. each. Express this weight in pounds.

4. An order is received in New Orleans for 8000 l. of molasses. About how many gallons is this?

Express the following approximately in our measures:

5. 2 l.	**11.** 2 km.	**17.** 6 m.	**23.** $\frac{1}{2}$ m.	**29.** 40 km.
6. 3 m.	**12.** $\frac{1}{2}$ km.	**18.** 2 cm.	**24.** 24 l.	**30.** 40 m.
7. 1 kg.	**13.** 4 km.	**19.** 20 m.	**25.** 10 l.	**31.** 40 l.
8. 5 l.	**14.** 8 km.	**20.** 8 l.	**26.** 10 m.	**32.** 0.5 m.
9. 5 kg.	**15.** 6 l.	**21.** 7 l.	**27.** 10 kg.	**33.** 0.5 km.
10. 1 km.	**16.** 9 l.	**22.** 7 m.	**28.** 10 cm.	**34.** 0.5 l.

Prefixes in the Metric System. Although we have now studied the most important measures in the metric system, we should know thoroughly the meaning of the prefixes used and learn the tables of length, weight, capacity, surface, and volume. It will be found that the metric system is very simple if these prefixes are known.

Just as 1 mill = 0.001 of a dollar,

so 1 millimeter = 0.001 of a meter.

Just as 1 cent = 0.01 of a dollar,

so 1 centimeter = 0.01 of a meter.

Just as the word decimal means tenths,

so 1 decimeter = 0.1 of a meter.

PREFIX	MEANS	AS IN	WHICH MEANS	
myria-	10,000	myriameter	10,000	meters
kilo-	1000	kilogram	1000	grams
hekto-	100	hektoliter	100	liters
deka-	10	dekameter	10	meters
deci-	0.1	decimeter	0.1	of a meter
centi-	0.01	centigram	0.01	of a gram
milli-	0.001	millimeter	0.001	of a meter

The teacher should show that this system is as much easier than our common one for measures and weights as the system of United States money is easier than the English system. This is the reason why the metric system is so extensively used on the continent of Europe and in Central America and South America.

It should always be remembered that measures never mean much to a student unless they are actually used in the schoolroom.

The students should be led to see that metric units may be changed to units of a higher or a lower denomination by simply moving the decimal point, exactly as in changing from 205¢ to $2.05.

Unless the student is to use the metric system in his measurements in science, the further study of this subject may be omitted.

Table of Length. The table of length is as follows:

A myriameter	=	10,000 meters
A **kilometer** (km.)	=	1000 meters
A hektometer (hm.)	=	100 meters
A dekameter	=	10 meters
Meter (m.)		
A decimeter (dm.)	=	0.1 of a meter
A **centimeter** (cm.)	=	0.01 of a meter
A **millimeter** (mm.)	=	0.001 of a meter

The meter is about 39.37 in., $3\frac{1}{4}$ ft., or a little over a yard; the kilometer is about 0.6 of a mile.

The names of the units chiefly used are printed in heavy black type in the tables.

The abbreviations in this book are recommended by various scientific associations. Some writers, however, use Km., Hm., cm. and mm. for kilometer, hektometer, centimeter, and millimeter.

Exercise 44. Length

Express as inches, taking 39.37 in. as 1 m.:

1. 35 m. **3.** 32 m. **5.** 275 cm. **7.** 5200 mm.

2. 60 m. **4.** 47.5 m. **6.** 4.64 cm. **8.** 8750 mm.

9. Express in inches the diameter of a 6-centimeter gun and the diameter of a 75-millimeter gun.

10. A certain hill is 425 m. high. Express this height in feet.

11. A tower is 48.6 m. high. Express this height in feet.

12. A wheel is 2.1 m. in diameter. Express this in inches.

13. The distance from Dieppe to Paris is 209 km. Express this distance in miles.

14. The distance between two places in Germany is 34.8 km. Express this distance in miles.

Table of Weight. The table of weight is as follows:

A **metric ton** (t.)	=	1,000,000 grams
A quintal (q.)	=	100,000 grams
A myriagram	=	10,000 grams
A **kilogram** (kg.)	=	1000 grams
A hektogram	=	100 grams
A dekagram	=	10 grams
Gram (g.)		
A decigram	=	0.1 of a gram
A centigram (cg.)	=	0.01 of a gram
A milligram (mg.)	=	0.001 of a gram

A kilogram, commonly called a *kilo*, is about $2\frac{1}{5}$ lb. A 5-cent piece weighs 5 g. A metric ton is about 2204.62 lb.

Table of Capacity. The table of capacity is as follows:

A **hektoliter** (hl.)	=	100 liters
A dekaliter	=	10 liters
Liter (l.)		
A deciliter	=	0.1 of a liter
A centiliter (cl.)	=	0.01 of a liter
A milliliter (ml.)	=	0.001 of a liter

A liter is the volume of a cube 1 dm. on an edge.

Exercise 45. Weight and Capacity

Express as kilos:

1. 244 lb.	**4.** 120 oz.	**7.** 2500 g.	**10.** 28.46 g.
2. 326 lb.	**5.** 68.4 lb.	**8.** 6852 g.	**11.** 5700 g.
3. 460 lb.	**6.** 400 T.	**9.** 252.5 lb.	**12.** 3268 g.

Express as liters, taking 1 l. as 1 qt.:

13. 5 hl.	**15.** 3.85 hl.	**17.** 4000 ml.	**19.** 656 hl.
14. 16 pt.	**16.** $37\frac{1}{2}$ gal.	**18.** 800 gal.	**20.** 9.65 hl.

Table of Square Measure. In measuring surfaces in the metric system we use the following table:

A square myriameter	=	100,000,000 square meters
A square kilometer (km^2.)	=	1,000,000 square meters
A square hektometer (hm^2.)	=	10,000 square meters
A square dekameter	=	100 square meters

Square meter (m^2.), about 1.2 sq. yd.

A square decimeter (dm^2.)	= 0.01	of a square meter
A square centimeter (cm^2.)	= 0.0001	of a square meter
A square millimeter (mm^2.)	= 0.000001	of a square meter

The abbreviation sq. m. is often used for m^2., sq. cm. for cm^2., and so on.

In measuring land the square dekameter is called an *are* (pronounced är); and since there are 100 square dekameters in 1 hm^2., a square hektometer is called a *hektare* (ha.). The hektare is equal to 2.47 acres, or nearly $2\frac{1}{2}$ acres.

Exercise 46. Square Measure

1. Find the area of a rectangle 3.2 m. by 12.7 m.

2. Find the area of a parallelogram whose base is 45 cm. and height 22.4 cm.

3. Find the area of a triangle whose base is 7.3 m. and height 4.6 m.

4. Find the area of each face of a cube of edge 27.2 cm.

5. A block of granite is 1.2 m. long, 0.8 m. wide, and 0.7 m. thick. Find the area of the entire surface.

6. A cylinder has a diameter of 0.75 m. and a height of 0.8 m. Find the area of each base: the area of the curved surface; the total area of the surface.

Table of Cubic Measure. In measuring volumes in the metric system we use the following table:

A cubic kilometer	= 1,000,000,000	cubic meters
A cubic hektometer	= 1,000,000	cubic meters
A cubic dekameter	= 1000	cubic meters

Cubic meter (m^3.), about $1\frac{1}{3}$ cu. yd.

A cubic decimeter (dm^3.)	= 0.001	of a cubic meter
A cubic centimeter (cm^3.)	= 0.000001	of a cubic meter
A cubic millimeter (mm^3.)	= 0.000000001	of a cubic meter

In measuring wood a cubic meter is called a *stere* (st.), but this unit is not used in our country.

Exercise 47. Cubic Measure

1. A box is 2.3 m. long, 1.7 m. wide, and 0.9 m. deep, interior measure. Find the volume.

2. What part of a cubic meter is there in a block of marble 1.3 m. long, 0.8 m. wide, and 0.6 m. thick?

3. A cubic centimeter of water weighs 1 g. How much does a cubic decimeter of water weigh? How much does a cubic meter of water weigh?

4. From Ex. 3 how much does 1 mm^3. of water weigh?

5. Knowing that gold is 19.26 times as heavy as water, find from Ex. 4 the weight of 1 mm^3. of gold.

6. A certain pile of wood is 7 m. long and 1.8 m. high, and the wood is cut in sticks 1 m. long. How many steres of wood are there in the pile?

Express the following as cubic meters:

7. 1250 dm^3.

8. 257,820 mm^3.

9. 2550 cm^3.

10. 2700 dm^3.

11. 50,200 dm^3.

12. 75,000 cm^3.

Exercise 48. Miscellaneous Problems

1. Write the formula for finding the volume of a cube, and also of a rectangular prism.

2. Write the formula for finding the area of each of the following: a circle; a trapezoid; a triangle; a parallelogram; a rectangle.

3. What must you have given and how do you proceed to find the area of a trapezoid? the volume of a cube? the area of a circle? the volume of a cylinder? the area of the curved surface of a cylinder? the area of a triangle? the area of the surface of a cube?

4. What must you have given and how do you proceed to find the number of cubic yards of earth to be removed in digging a cellar in the shape of a rectangular solid?

5. What must you have given and how do you proceed to compute the cost of polishing a cylindric marble monument at 52¢ a square yard?

6. How many loads of gravel averaging 1 cu. yd. each will be required to grade $2\frac{1}{4}$ mi. of road, the gravel to be laid 15 ft. wide and 6 in. deep?

7. A water tank is 7 ft. 6 in. long and 5 ft. 9 in. wide. Water is flowing through a pipe into the tank at the rate of 3 cu. ft. in 2 min. How long will it take to fill the tank to a depth of 3 ft. 8 in.?

8. At what heights on the sides of a cylindric measuring vessel whose base is 8 in. in diameter should the marks for 1 gal. (231 cu. in.), 1 qt., 2 qt., and 3 qt. be placed?

9. Water is flowing into a cylindric reservoir 28 ft. in diameter at the rate of 280 gal. a minute. Find the rate, that is, the number of inches per minute, at which the water rises in the reservoir.

Exercise 49. Optional Outdoor Work

1. Make the necessary measurements; then draw a plan of your school grounds to a convenient scale. Indicate on the plan the correct position and the outline of the ground covered by the school building.

2. Draw a plan of the athletic field where your school games are played. Indicate on the plan the correct positions of the principal features of the field.

3. Step off distances of 100 ft. in various directions on your school grounds. Check the distance each time before stepping off the next. Do the same for distances of 75 ft., 140 ft., and 60 ft.

4. Estimate the distance between two trees or other objects on your school grounds, then step off the distance. Check by measuring the distance.

5. Estimate the height of several objects on or near your school grounds, and then check the accuracy of your estimate by determining the heights of the objects by some of the methods previously explained.

6. Estimate the number of square rods or acres in your school grounds. Check your estimate by measurement.

7. Estimate the number of acres in one of the city or village blocks near your school or near your home. Check your estimate by measurement.

8. Lay off, by estimate, on your school grounds an area which you believe is a quarter of an acre. Check your work by measurement.

9. Estimate the extreme length and the height of your school building. Check your estimates. Which is the more difficult for you to estimate with a fair degree of accuracy, length or height?

Exercise 50. Problems without Figures

1. If you know the dimensions of a field in rods, how do you find the area in acres? If you know the dimensions in feet, how do you find the area in acres?

2. If you can make all necessary measurements of a triangular field, how can you find the area?

3. If you can make all necessary measurements of a trapezoidal field, how can you find the area?

4. How can you determine the width of a stream without crossing the stream?

5. How can you determine the height of a church spire without climbing to the top?

6. If you have a good map of the state of Colorado, drawn to a scale which you know, how can you find from the map the number of square miles in the state?

For our present purposes we may consider that the state is rectangular, although this is only an approximation.

7. If you know the circumference of a circular water tank, how can you find the diameter?

8. If you know the circumference of a circle, how can you find the area?

Consider first the result of Ex. 7.

9. If you know the circumference and height of a cylindric water tank, how can you find the capacity?

Consider Exs. 7 and 8.

10. If you know the length of a water pipe in feet and the internal diameter of the pipe in inches, how can you find the number of cubic inches of water it will hold? How can you then find the number of gallons and the weight of the water it will hold?

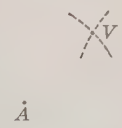
III. GEOMETRY OF POSITION

Position of Objects. We have already seen, on page 111, that geometry can ask three questions about an object: (1) What is its shape? (2) How large is it? (3) Where is it? The first question led us to study those common forms about which everyone needs to know; the second led us to find the size of such forms, including not only areas and volumes but also heights and distances; and now we come to the third question and consider position.

For example, when two boys have located the home plate and second base in laying out a baseball field, how can they find the positions of first base and third base or the position of the pitcher's box? Or if they have located the home plate and the line to the pitcher's box, how can they locate the other bases? Such questions show that position is an important subject in geometry.

It has often happened in war that people have buried their valuables. They did not dare to mark the spot, because then their enemies might find the hiding place and dig up the valuables; and so it became necessary to be able to locate the spot in some other way.

This may be done in various ways. For example, a man may select two trees, A and B, 160 ft. apart, and measure the distance from each tree to V, the point where he buried his valuables, say 80 ft. and 120 ft., remembering that V is north of a line drawn from A to B. Then when he returns he may take two pieces of rope 80 ft. and 120 ft. long, besides what is needed for tying, and by stretching these from the trees A and B respectively he can find V in just the same way that the triangle on page 116 was constructed.



Exercise 51. Fixing Positions

1. During a war a man buried some valuables. He remembered that they were buried south of an east-and-west line joining two trees 60 ft. apart, and that the point was 50 ft. from the eastern tree and 70 ft. from the western tree. Draw a plan to the scale of 10 ft. = 1 in. and indicate the point where the valuables were buried.

2. In olden times, before there were many banks, it was the common plan to bury treasures for safe-keeping. Suppose that a man buried a treasure 120 ft. from one tree and 160 ft. from another one, but that during his absence the second tree is entirely destroyed. Draw a plan showing where he should dig a trench so as to strike the treasure.

3. Two points, A and B, are 3 in. apart. Is it possible to find a point which is 1 in. from each? $1\frac{1}{2}$ in. from each? 2 in. from each? 3 in. from each? Draw a figure illustrating each case and consider whether, in any of the cases, there is more than one point.

4. To keep the water clean a farmer covers a spring with a large flat stone and pipes the water to his house. He then covers the stone and piping with soil, and seeds it all down. Before doing this he takes the necessary measurements for locating the spring, using as the known points two corners of the field in which it is. Draw three plans showing how to take the measurements.

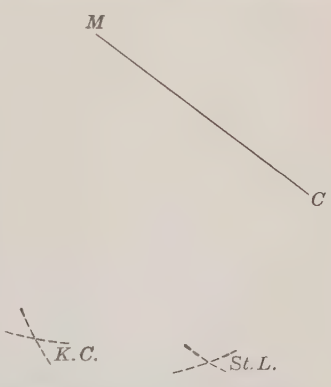
5. Two streets, each 66 ft. wide, intersect at right angles. Two water mains meet 40 ft. from a certain corner and 45 ft. from the diagonally opposite corner. Some workmen wish to dig to find where the mains meet. Draw a plan to scale and show the two possible places where they should dig.

Positions on Maps. Knowing the shortest distance from Chicago to Minneapolis to be approximately 354 mi., that from Chicago to St. Louis 263 mi., that from Chicago to Kansas City 415 mi., that from Minneapolis to St. Louis 466 mi., and that from Minneapolis to Kansas City 411 mi., we can easily make a map showing the location of all four of these cities.

If we draw the map to the scale of 1 in. = 252 mi., the map distance from Chicago to Minneapolis will be $\frac{354}{252}$ in., or 1.40 in., and similarly for the other distances. That is,

Chicago to Minneapolis	$\frac{354}{252}$ in. = 1.4 in.
Chicago to St. Louis	$\frac{263}{252}$ in. = 1.0 in.
Chicago to Kansas City	$\frac{415}{252}$ in. = 1.6 in.
Minneapolis to St. Louis	$\frac{466}{252}$ in. = 1.8 in.
Minneapolis to Kansas City	$\frac{411}{252}$ in. = 1.6 in.

With the aid of a ruler draw the line CM, making it 1.4 in. long, C representing Chicago and M representing Minneapolis. Next place the compasses with one point at C and draw an arc with a radius 1.0 in., representing the distance to St. Louis; also draw an arc with radius 1.6 in., representing the distance to Kansas City. Similarly, with center M and radii 1.8 in. and 1.6 in. describe arcs intersecting the other arcs, thus locating St. Louis and Kansas City. We can now approximate the distance from Kansas City to St. Louis, for if it is 0.95 in. on the map, it is really 0.95 of 252 mi., or 239 mi.



Exercise 52. Map Drawing

1. The distance from Cincinnati to Cleveland is 222 mi., from Cincinnati to Pittsburgh 258 mi., and from Cleveland to Pittsburgh 115 mi. Draw a line to represent the distance from Cincinnati to Cleveland and then mark the position of Pittsburgh. Use the scale of 1 in. to 50 mi.

It is to be understood that the distances stated in this and the following problems are only approximate, and that they are measured in a straight line and not along roads or railways.

2. The distance from Philadelphia to Harrisburg is 95 mi., from Harrisburg to Baltimore 70 mi., and from Baltimore to Philadelphia 90 mi. Indicate the position of the three cities, using the scale of 1 in. to 75 mi.

3. On a map drawn to the scale of 1 in. to 108 mi. the distance from Atlanta to Raleigh is $3\frac{1}{8}$ in., that to Savannah 2 in., and that to Jacksonville $2\frac{7}{16}$ in. What is the distance in miles from Atlanta to each of the other cities?

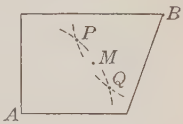
4. On a map of scale 1 in. = 145 mi. the distance from San Francisco to Portland is $3\frac{3}{4}$ in., that to Seattle $4\frac{7}{8}$ in., and that to Los Angeles $2\frac{1}{4}$ in. What is the distance in miles from San Francisco to each of the other cities?

5. On a map drawn to the scale of 1 in. to 95 mi. the extreme length of Pennsylvania is 3 in. and the extreme width is $1\frac{5}{8}$ in. What are the dimensions of the state? What is the approximate area of Pennsylvania?

6. The distance from Nashville to Memphis is $2\frac{1}{16}$ in. measured on a map drawn to the scale of 1 in. to 90 mi. The distance from Nashville to Louisville is $1\frac{11}{16}$ in., from Nashville to Mobile $4\frac{1}{8}$ in., and from Nashville to Chattanooga $1\frac{1}{8}$ in. What is the distance in miles from Nashville to each of the other cities?

Points Equidistant from Two Points. We shall now consider again the case of a spring of water that has been covered by a stone slab and earth so that its position is not visible. If the farmer who owns the land has lost the measurements and remembers only that the spring was just as far from one corner A of the field as from the diagonally opposite corner B, how shall he dig to find the spring?

If A and B are 540 ft. apart he might try digging at M, the mid-point between them, 270 ft. from both A and B. Failing here, he might try a point P, which is 280 ft. from both A and B. He might then try a point Q, which is also 280 ft. from both A and B, and so on. He might thus try a large number of points, and yet miss the spring. It would be better for him to run a line along the ground and know that if he should dig along this line he would certainly strike the spring. Let us see how this could be done.



If you connect P, M, and Q, what kind of line do you seem to have? What does this suggest as to how the farmer should run the line?

Take two points A and B on paper; find four points such that each is equidistant from A and B, and then connect these points and see if you have the kind of line you expect. Was it necessary to take as many as four points? How many points need be taken?

From the above work we see the truth of the following:

To find all the points which are equidistant from two given points, find any two such points and draw a straight line through them. All the required points lie on this line or on the prolongation of this line.

Instead of using paper or blackboard for the above work, points may be taken on the schoolroom floor or on the school grounds.

Exercise 53. Points Equidistant from Two Points

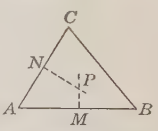
1. Two water pipes are known to join at a place equidistant from two trees which are 160 ft. apart. One tree is exactly north of the other, and the pipes join west of the line connecting them. Draw a plan on the scale of 1 in. to 40 ft. and show the line along which a workman should dig to find where the pipes join.

2. The residences of Mr. Anderson and Mr. Williams are 300 ft. apart and on the same side of a straight water main. Mr. Anderson's residence is 50 ft. and Mr. Williams's residence is 65 ft. from the water main. They decide to have trenches dug so as to strike the main at a point equidistant from the two houses. Draw a plan on the scale 1 in. = 60 ft. showing the position of the trenches.

3. It is planned to place a circular flower bed in a park so that its center shall be 90 ft. distant from each of two trees which are 80 ft. apart. The radius of the flower bed is to be 20 ft. Draw a plan on the scale of 1 in. to 20 ft.

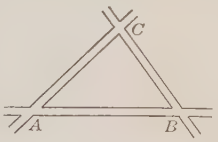
4. How can you find a point on one side of your schoolroom which is equidistant from two of the diagonally opposite corners? Draw a plan of your schoolroom on the scale of 1 in. to 10 ft. and locate the point on the side of the room.

5. A landscape gardener wishes to plant a tree which shall be equally distant from three trees forming the vertices of a triangle, as shown in the figure. How can he find where to plant the fourth tree? Suppose it is known that $AB = 140$ ft., $BC = 110$ ft., and $CA = 130$ ft., draw a plan on the scale of 1 in. to 20 ft. and determine the location of the fourth tree and its distance from each of the other trees.

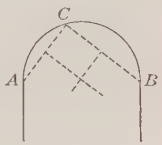


6. Does a point equally distant from the three vertices of a triangle always lie inside the triangle? Consider these triangles: $AB = 4$ in., $BC = 7$ in., $CA = 6$ in.; $AB = 3$ in., $BC = 4$ in., $CA = 5$ in.; $AB = 2$ in., $BC = 3$ in., $CA = 1.5$ in. Construct the triangles and locate the points.

7. In a park there are three walks which meet as shown in the figure, and $AB = 180$ ft., $BC = 125$ ft., and $CA = 145$ ft., inside measure. It is planned to place a cluster of lights which shall be equally distant from the three corners A, B, and C. Draw a plan to some convenient scale, indicate the position at which the cluster of lights is to be placed, and determine its distance from each of the three corners.



8. In an athletic park are three trees A, B, and C, so situated that $AB = 150$ ft., $BC = 115$ ft., and $CA = 90$ ft. A running track is planned, as shown in the figure, so that an arc of a circle shall pass close to the trees. Draw a plan to any convenient scale and determine the radius of the circle.



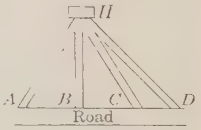
9. In the ruins of Pompeii a portion of the tire of an old chariot wheel was found. The curator of a museum wished a drawing of the original wheel. How was it possible to determine the center and to reproduce the tire of the wheel in a drawing?

10. Three points, A, B, and C, are so situated that $AB = 3.5$ in., $BC = 2.5$ in., and $CA = 4.5$ in. By drawing to scale and measuring, determine the length of the radius of the circle which passes through the three points.

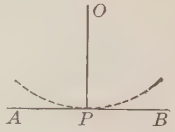
11. Solve Ex. 10 for three points which are so situated that $AB = 4$ ft. 2 in., $BC = 4$ ft. 8 in., and $CA = 5$ ft. 1 in.

Distance of a Point from a Line. In the figure below, if H represents a house and AB a straight road, how far is it to the house from the road?

It is obvious that the answer to the question depends on how we are to go. If there are several straight paths such as HA, HB, HC, and HD leading to the house from the road, the length of each path is an answer to the question. We see at once, however, that the shortest of the paths shown in the figure is HB, and when we speak of the distance from the house to the road or from a point to a line, we mean the shortest distance.



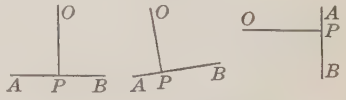
If a stone P is hanging by a string and the other end of the string is held at O, and if the stone swings so as just to graze the line AB, it is evident that OP represents the shortest distance from O to the line and also that OP makes two right angles with AB. These two right angles are BPO and OPA, and; of course, are equal.



Perpendicular. There is a special name for the straight line which represents the shortest distance from a point to a line. In the above figure OP is said to be *perpendicular* to AB or to be *the perpendicular* from O to AB.

The perpendicular from a point to a line makes right angles with the line and is the shortest path from the point to the line.

Perpendicular must not be confused with vertical. A plumb line (a line with a weight at the end) hangs vertically, but a line may be perpendicular to another line and still not be vertical. In each of these figures OP is perpendicular to AB, but in only the first of the figures is OP vertical.



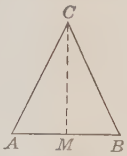
Exercise 54. Distance of a Point from a Line

1. Draw a perpendicular to the line AB from the point O outside the line, proceeding as follows: Set one point of the compasses at O, then adjust the compasses until the other point just grazes the line AB at P. Draw OP; then OP is evidently perpendicular to AB.

See the second figure on page 222.

This method of drawing the perpendicular shows clearly what the perpendicular is, but it is not frequently used in practice. Usually the best method is that which is described on page 121 or that on page 122.

2. In this figure AB is 45 ft., and AC and BC are each 52 ft. Find by measurement the perpendicular distance CM.

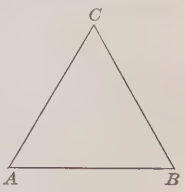


3. A straight gas main runs from one street light A to another light B, the distance AB being 220 ft. It is desired to place a third lamp C in a park fronting on the street, so that the distance AC shall be 260 ft. and the distance BC shall be 190 ft. The gas main AB is to be tapped at the point nearest to C.

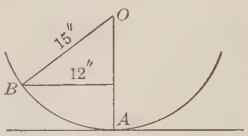
Draw the figure to the scale of 1 in. to 40 ft.; find by measurement the approximate distance of C from AB and the distance from B to the point where the main is tapped.

The figure is somewhat like that of Ex. 4.

4. In this figure ABC is an equilateral triangle each of whose sides is 14 in. Draw the triangle to a convenient scale and determine the length of the perpendiculars drawn from A to BC and from B to AC. Does the perpendicular drawn from C to AB appear to pass through the point in which the other two perpendiculars intersect?



5. A stone is tied to a string 15 in. long. The string is held at the other end, and the stone is then swung so that it just grazes the ground. When the stone is at the point B, 12 in. from the perpendicular OA, how far is it from the ground? Draw the figure carefully to some convenient scale and thus find this distance by measurement.



6. Some boys stretch cords from the top of a flagstaff to two points on level ground 40 ft. apart. The first cord is 90 ft. long and the second is 65 ft. long. Draw the figure to scale and thus find the height of the flagstaff.

7. Two boys observe a bird on the top of a 90-foot flagstaff. One boy is 65 ft. from the foot of the staff and the other is 130 ft. from its foot. Draw the figure to scale and thus find the distance of each boy from the bird.

8. Two trees on the same bank of a straight river are 285 ft. apart. The distance from one of the trees to a point on the opposite bank is 200 ft. and the distance from the other tree to the same point is 260 ft. Draw the figure to scale and thus find the width of the river.

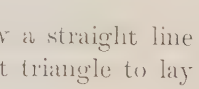
9. An airplane is 2800 ft. above a level railway line connecting two villages A and B, which are 1.5 mi. apart. The airplane is above a point which is $\frac{3}{4}$ of the distance from A to B. Draw the figure to scale and thus find the distance of the airplane from each of the villages.

10. A pyramid has a square base 120 ft. on a side. The distance from the vertex to the mid-point of one side is 90 ft. Draw the figure to scale and thus find the height of the pyramid.

Points at a Stated Distance from a Line. If a man wishes to build a house at a distance of 100 ft. from a straight road, he can build it in any one of a great many places, for he can stand anywhere on the road, lay off a perpendicular on either side of it, and then measure a distance of 100 ft. along this perpendicular, any point thus found being 100 ft. from the road.

Pages 121 and 122 may profitably be reviewed at this time.

To illustrate this further we may draw a straight line XY, using a carpenter's square or a right triangle to lay off four points on one side of the line and $\frac{1}{4}$ in. from it and to lay off four points on the other side of the line and $\frac{1}{4}$ in. from it. It is evident that the points lie in two lines each of which is $\frac{1}{4}$ in. from XY. All points on the paper $\frac{1}{4}$ in. above or below XY evidently lie on one of these two lines.



In geometric and mechanical drawing it is necessary to be able to locate such points quickly and accurately.

To draw a line perpendicular to a given line XY, we may, as stated on page 121, place a right triangle ABC so that one side BC lies along XY and the hypotenuse AC along a ruler MN. Then AB is perpendicular to XY, and as we slide the triangle along MN, as shown in the figure, AB will remain perpendicular to XY and the line can be drawn with accuracy right across XY.

After we have drawn two such perpendiculars on each side of XY, and the required distance has been laid off, we can draw the two lines which contain all the points at the given distance from the line. It is evident that we need to find only two points of a straight line in order to draw the line with a ruler.

Exercise 55. Points at a Given Distance from a Line

1. Draw a plan showing 40 ft. of a straight railroad track 4 ft. $8\frac{1}{2}$ in. wide, using the scale of 1 in. to 8 ft. and being careful to make the two rails everywhere equally distant from each other.

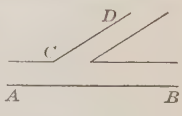
2. Using a convenient scale, draw a plan of the straight-away of a running track showing 100 yd. of a straight track 16 ft. wide.

3. Draw a plan of the floor of your schoolroom, being careful to make the sides perpendicular at each corner. If the opposite sides are parallel you can test whether your schoolroom is a perfect rectangle by measuring the diagonals to find whether they are equal.

4. An old description of a corner lot states that there is a covered well 30 ft. from one of two streets which cross at right angles (but it does not state which street) and 50 ft. from the other street. Draw a plan to a convenient scale and indicate all possible positions of the well.

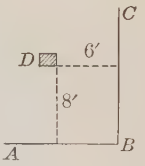
5. The residence of Mr. Weber is 130 ft. from a straight road. He wishes to build a garage 80 ft. from his house and 120 ft. from the road. Indicate on a plan the position of the house and the garage.

6. Two straight driveways in a park meet as shown in the figure. A fountain is to be placed 80 ft. from driveway AB and 60 ft. from driveway CD. Draw a figure to scale showing the two possible positions of the fountain and determine the distance between these positions. First copy the figure accurately, enlarging it on any convenient scale, extending it as may be necessary, and taking the width of driveway AB to be 40 ft.



Position fixed by Two Lines. Having already seen how the position of an object can be found if we know its distance from each of two points, let us see if we can find the position if we know its distance from each of two lines.

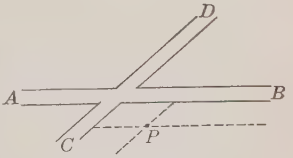
Take, for example, your desk. If anyone was told that your desk is 6 ft. to the west of the east wall and 8 ft. north of the south wall, could he find which desk is yours? How would he do it? In this plan he might run a line 6 ft. from BC and parallel to it, and another line 8 ft. from AB and parallel to it, and where these lines crossed he would find the desk D.



This is the way we locate a place on a map of the world. We say it is so many degrees north or south of the equator and so many degrees east or west of the meridian of Greenwich. Thus we say that a place is 40° N. and 70° W., meaning that it is 40° north of the equator and 70° west of the prime meridian, the one which passes through Greenwich.

The principle applies whether we use a Mercator's projection or a globe, except that the curvature of the lines is seen in the latter case.

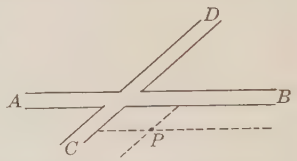
The lines we use need not meet at right angles. For example, if we know that two water mains join at a point 60 ft. south of the road AB, as here shown, and 90 ft. from the road CD and on the side towards B, we can locate the point P by simply drawing two lines. How are these lines drawn?



If we know that P is 60 ft. south of AB and 90 ft. from CD, but do not know on which side, there would be two possible points. Under what circumstances would there be three possible points, or four possible points?

Exercise 56. Position fixed by Two Lines

1. Suppose that we do not know the side of the road CD on which the two water mains mentioned on page 227 join, but we do know that it is on the south side of the road AB. Draw a figure showing all the possible positions of the water mains.



2. Suppose that we know that the water mains join to the west of CD, but do not know whether the point is to the north or to the south of AB. Draw a figure showing all the possible positions.

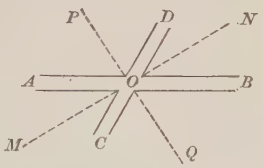
3. Suppose that we are uncertain as to the side of each road on which the water mains join. Draw a figure showing all the possible positions.

4. Each side of a square park $ABCD$ is 150 yd. long. A monument is to be erected in the park at a point 130 ft. from AB and 170 ft. from BC. Draw the figure to scale and determine the distance of the monument from each corner of the park.

5. There are three survivors of a shipwreck. The first says that the ship lies between 2 mi. and $2\frac{1}{2}$ mi. from a straight coast line which runs from a lighthouse L to the west; the second says that the ship lies between $1\frac{1}{2}$ mi. and 2 mi. from a straight coast line which runs from L to the north; and the third says that it lies $2\frac{1}{2}$ mi. from L. Can they all be right? If so, draw the figure to scale and indicate where to dredge for the wreck.

6. In a rectangular field $ABCD$, 90 rd. by 130 rd., there is a water trough which is 290 ft. from the side AB and 320 ft. from BC. Draw the figure to scale and thus find the distance of the water trough from the point D.

Points Equidistant from Two Lines. Suppose that two roads AB and CD intersect at O and that it is desired to place a street lamp at a point equidistant from the two streets. How many possible positions are there for the lamp?

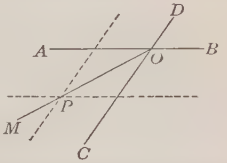


If we think of ourselves as walking in such a direction as to be always equidistant from OB and OD, we see that we shall be walking along ON. Similarly, to be equidistant from OD and OA we must walk along OP. In general, any point on any of the dotted lines in the figure is equidistant from AB and CD. Therefore we may locate the lamp anywhere on either line, these lines evidently bisecting the angles formed by the roads.

Exercise 57. Points Equidistant from Two Lines

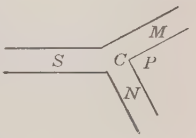
1. By using this figure draw a line containing points equidistant from two lines.

The dotted lines are each $\frac{1}{4}$ in. from AB and CD respectively. They intersect at P, and OP is drawn and prolonged to M.



2. Draw a line containing points equidistant from two lines by using a figure somewhat like the one used in Ex. 1, page 228.

3. In a park P, which lies between two streets M and N, as shown in the figure, an electric light is to be placed so as to be equidistant from M and N and 80 ft. from the corner C. Copy the figure and show how to find the position of the light.

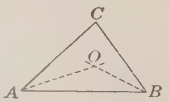


In this case a circle intersects a straight line.

4. The manager of an amusement park decides to place six lights at intervals of 80 ft., so that each light shall be equidistant from two intersecting driveways M and N, as shown in the figure. The first light is to be placed 15 ft. from the intersection of the driveways. Copy the figure to scale and indicate the position of each of the six lights.

5. A fountain is to be erected in the space S between the two streets M and N. The contract provides that the fountain shall be 50 ft. from the nearest side of each of these streets. Copy the figure and indicate the position of the fountain.

6. A monument is to be so placed in a triangular city park that it shall be equally distant from the three sides. Draw a plan on any convenient scale and show on the plan all the lines necessary to find the point O. Is O equidistant from the sides AB and BC? Is it equidistant from AB and CA? Is it necessary to draw a line from C?

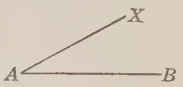


7. A flagpole is to be so placed that it shall be equally distant from the three sides of a triangular park whose sides are 380 ft., 270 ft., and 300 ft. respectively. Draw a plan on a convenient scale and find the distance of the flagpole from each side of the park.

8. A water main has a gate located at a point 7 ft. from a certain lamp-post which stands on the edge of a straight sidewalk. The gate is placed 3 ft. from the edge of the walk, towards the street. Draw a plan showing every possible position of the gate.

9. Consider Ex. 8 when the gate is located at a point 3 ft. from the lamp-post.

Use of Angles. There is still another method by which a man may locate valuables which he has buried. Suppose as before that there are two trees, A and B, and that he has buried his valuables at X. If he knows the exact direction and distance from A to X, he can easily find the place where the valuables are buried.

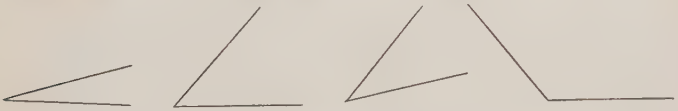


Both the distance and direction can be recorded on paper. For example, the man might write: "30° north of line joining the trees, 150 ft. from the west tree," and this would recall to his mind that the angle of 30° is to be measured at A, namely, the angle BAX, and that X will be found 150 ft. from A. The man can lay off the angle on a piece of paper by the aid of a protractor like the one described on page 115, and can then sight along the arms of this angle.

The use of the protractor, as given on page 115, may be reviewed at this time if necessary.

Exercise 58. Uses of Angles

1. Estimate the number of degrees in each of the following angles, then check your estimate by use of a protractor.



2. Without using a protractor draw several angles of as near 45° as you can, and in different positions. Check the accuracy of your drawings by measuring the angles with a protractor. Similarly, draw angles of 10°, 80°, 75°, 15°, 50°, and 30°. Check the accuracy of your drawing in each case.

You will find that practice will enable you to estimate lengths and the size of angles with a fair degree of accuracy.

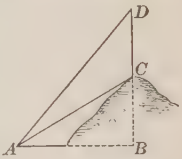
3. Indicate on paper a direction 30° west of south of the school; 20° east of north; 20° east of south.

By 30° west of south is meant a direction making an angle of 30° with a line running directly south, and to the west of that line.

4. It is found that a submarine cable is broken 5 mi. from a certain lighthouse and 30° west of south. Draw a plan, using the scale of 1 in. to 1 mi., and show where the repair ship must grapple to bring up the ends for splicing.

5. A boy starts to walk on a straight road which runs 20° east of north. After he has gone 3 mi. he turns on another straight road and walks 2 mi. due west. Is he then east or west of his starting point, and how many degrees?

6. A flagstaff CD stands on the top of a mound the height BC of which is known to be 30 ft. From the point A the angle BAD is observed to be 50° and the angle CAD to be 20°. Draw a diagram to scale and find approximately the height of the flagstaff CD.

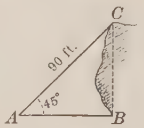


7. Wishing to find the length of a pond, some boys staked out a right triangle as shown in the figure. Using a protractor they found that the angle A was 40°, and they measured AB, finding it to be 500 ft. Draw the figure to the scale of 1 in. to 200 ft. and thus find the length of the pond.

8. Some men buried a chest 150 yd. from a tree A and 250 yd. from another tree B, which was 200 yd. due north of A. On returning some years later they found that the tree B had disappeared. Draw a plan to scale and show how they can find the buried chest, provided that they have a compass and remember on which side of AB they hid the chest.

9. A seaport is on a straight coast which runs due north and south. A steamer sails from it in a direction 30° west of north at the rate of 15 mi. an hour. When will she be 25 mi. from the coast, and how far will she be from the seaport at that time?

10. Some boys wished to determine the height of a cliff. They found the distance AC to be 90 ft. and the angle BAC to be 45°. They then drew the figure to scale and determined the height of the cliff. What is the height of the cliff?

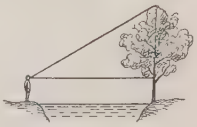


11. While a ship is steaming due east at the rate of 20 knots an hour, the lookout observes a light. At 9 P.M. the light is due north, at 9.15 P.M. it is 10° west of north, and at 9.30 P.M. it is 20° west of north. Determine by a drawing whether the light observed is stationary.

A knot is generally taken as 6080 ft., this being approximately the sea mile; the statute mile used on land is 5280 ft.

12. There is a seaport A on a straight coast running east and west. A rock B lies 3000 yd. from A and 30° north of east from the coast line. A ship steams from A in a direction 20° north of east from the coast line. What is the nearest approach of the ship to the rock? How far is the ship from the coast at that time?

13. A man knows a tree across a stream to be 60 ft. high; he finds the angle at his eye as shown in the figure to be 30°, and he knows that his eye is 5 ft. from the ground. Make a drawing to scale and determine the width of the stream.



14. Two towers of equal height are 300 ft. apart. From the foot of each tower the top of the other tower makes an angle of 30° with a horizontal line. Determine the height of the towers.

Exercise 59. Miscellaneous Problems

1. A ship steams from a seaport A in a direction 50° east of north. A dangerous rock lies northeast from A and 5000 yd. from the coast running east and west through A. Find how near the ship approaches to the rock.

2. Two sides of a rectangular field $ABCD$ are 80 rd. and 95 rd. A tree in the field stands 120 ft. from AB and 160 ft. from BC. Draw the figure to scale and show the position of the tree.

3. A man computes that he should buy 16 T. of soft coal for his winter supply. His cellar is so arranged that he can make a coal bin 18 ft. long, and the height of the cellar is $8\frac{1}{2}$ ft. How wide a bin should be constructed if the top of the coal is to be a foot from the ceiling when the bin contains 16 T. of coal?

Allow 35 cu. ft. of coal to the ton.

4. A boy standing due south of a flagpole finds its angle of elevation, that is, the angle to the top, to be 20°. After he has walked 280 ft. to the northwest on level ground, he sees the flagpole to the northeast. Draw the figure to scale and determine the height of the flagpole.

5. How long a shadow will the flagpole mentioned in Ex. 4 cast when the sun's angle of elevation is 40°?

6. A tree 90 ft. high casts a shadow 140 ft. long. Draw the figure to scale and find the sun's angle of elevation.

7. A military commander standing 1000 ft. from a fort finds its angle of elevation to be 30°. Draw the figure to scale and determine the height of the fort.

8. In order to check his work in Ex. 7 the commander took the angle of elevation from a point 1200 ft. from the fort. What did he find this angle to be?

Exercise 60. Optional Outdoor Work

1. Determine the height of a tree on or near your school grounds by finding the angle of elevation of the top from two different positions and measuring the distance of each position from the tree.

2. Two boys wishing to determine which of two smokestacks is the taller computed the height of each stack by three different methods and then took the average of the results as the correct height. What three methods might they have used? Use three methods to determine the height of some high object near your school.

3. A graduating class decides to present a drinking-fountain to the school. The fountain is to be placed 30 ft. from a straight street which runs in front of the school grounds and 15 ft. from the front door of the school building. How could the members of the class determine the desired position? Locate, if possible, such a position on your school grounds or on some lot in the vicinity.

4. Suggest two methods of determining the distance between two points when the distance cannot be measured directly. Use both methods to determine the distance between two easily accessible points on your school ground. Check the accuracy of each method by actually measuring the required distance.

5. A hawk's nest is observed in a high tree and some distance below the top of the tree. Suggest two methods by which the height of the nest may be determined other than by direct measurement. Check the accuracy of the two methods by applying them to a similar situation where the required distance can be actually measured as a check upon the accuracy of the methods.

Exercise 61. Problems without Figures

1. A workman has a circular disk of metal and wishes to find its exact center. How should he proceed?

2. A man wishes to set out a tree so that it shall be equally distant from three trees which are not in the same straight line. How should he proceed to find the position?

3. A contractor wishes to tap a water main at a point equidistant from two hydrants. How should he proceed to find the required point?

4. A city engineer is asked to place an electric-light pole at a point equidistant from two intersecting streets and at a given distance from the corner. How does he do it?

5. A man wishes to build on a corner lot a house at a given distance from one street and at another given distance from the other street. How does he lay out the plan?

6. A drinking-fountain is to be placed in a park at a given distance from a hydrant which is at the side of the street in front of the park, and at a given shorter distance back from the street. Show that there are two possible points, and show how to find them.

7. If you know that two water mains join somewhere under a certain road, but you do not know where, what measurements could be given you with respect to one or more trees along the side of the road that would tell you where to dig to find the point?

8. A straight electric-light wire runs under the floor of a room. The distances of one point of the wire from the northeast corner and from the north wall are known, and also the distances of another point from the southwest corner and from the south wall. Show how to mark on the floor the course of the wire.

IV. SUPPLEMENTARY WORK

Squares and Square Roots. If a square has a side 4 units long, it has an area of 16 square units. Therefore 16 is called the *square* of 4, and 4 is called the *square root* of 16.

Square Roots of Areas. Therefore, considering only the abstract numbers which represent the sides and area,

The side of a square is equal to the square root of the area.

Symbols. The square of 4 is written 4^2, and the square root of 16 is written $\sqrt{16}$.

Perfect Squares. Such a number as 16 is called a *perfect square*, but 10 is not a perfect square. We may say, however, that $\sqrt{10}$ is equal to 3.16 +, because 3.16^2 is very nearly equal to 10.

Square Roots of Perfect Squares. Square roots of perfect squares may often be found by simply factoring the numbers.

For example,
$$\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7}$$
$$= \sqrt{3 \times 7 \times 3 \times 7}$$
$$= \sqrt{21 \times 21} = 21.$$

$$\begin{array}{r|l} 3 & 441 \\ 3 & 147 \\ 7 & 49 \\ \hline & 7 \end{array}$$

That is, we separate 441 into its factors, and then separate these factors into two equal groups, 3×7 and 3×7. Hence we see that 3×7, or 21, is the square root of 441.

We prove this by seeing that $21 \times 21 = 441$.

To find the square root of a perfect square, separate it into two equal factors.

The work in square root may be omitted if there is not time for it.

Square of the Sum of Two Numbers. Since $47 = 40 + 7$, the square of 47 may be obtained as follows:

$$
\begin{array}{rl}
 & 40 + 7 \\
 & 40 + 7 \\
\hline
 & (40 \times 7) + 7^2 \\
40^2 + & (40 \times 7) \\
\hline
40^2 + 2 \times (40 \times 7) + & 7^2 \\
= 1600 + 2 \times 280 + & 49 \\
= 1600 + 560 + & 49 \\
= 2209. &
\end{array}
$$

280	49
1600	280
40	7
t	$+ u$

This relationship is conveniently seen in the above figure, in which the side of the square is $40 + 7$.

Every number consisting of two or more figures may be regarded as composed of tens and units. Therefore

The square of a number contains the square of the tens, plus twice the product of the tens and units, plus the square of the units.

This important principle in square root should be clearly understood, both from the multiplication and from the illustration.

Separating into Periods. The first step in extracting the square root of a number is to separate the figures of the number into groups of two figures each, called *periods*.

Show the class that $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so it is evident that the square root of any number between 1 and 100 lies between 1 and 10, and the square root of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number expressed by *one* figure or *two* figures is a number of *one* figure; the square root of any integral number expressed by *three* or *four* figures is a number of *two* figures; and so on.

Therefore, if an integral number is separated into periods of two figures each, from the right to the left, the number of figures in the square root is equal to the number of the periods of figures. The last period at the left may have one figure or two figures.

Extracting the Square Root. The process of extracting the square root of a number will now be considered, although in practice such roots are usually found by tables.

For example, required the square root of 2209.

Show the class that if we separate the figures of the number into periods of two figures each, beginning at the right, we see that there will be two integral places in the square root of the number.

The first period, 22, contains the square of the tens' number of the root. Since the greatest square in 22 is 16, then 4, the square root of 16, is the tens' figure of the root.

Subtracting the square of the tens, the remainder contains twice the tens × the units, plus the square of the units. If we divide by twice the tens (that is, by 80, which is 2 × 4 tens), we shall find approximately the units. Dividing 609 by 80 (or 60 by 8), we have 7 as the units' figure.

```
  22 09 ( 47
  16
 ------------
 80 | 6 09
 87 | 6 09
 ------------
```

Since twice the tens × the units, plus the square of the units, is equal to (twice the tens + the units) × the units, that is, since $2 \times 40 \times 7 + 7^2 = (2 \times 40 + 7) \times 7$, we add 7 to 80 and multiply the sum by 7. The product is 609, thus completing the square of 47. Checking the work, $47^2 = 2209$.

Exercise 62. Square Root

Find the square roots of the following numbers:

1. 3249. **2.** 3721. **3.** 3969. **4.** 5041.

Find the sides of squares, given the following areas:

5. 6724 sq. ft. **7.** 9025 sq. ft. **9.** 7921 sq. yd.

6. 7569 sq. ft. **8.** 9409 sq. ft. **10.** 6889 sq. ft.

Find the square roots of the following fractions by taking the square root of each term of each fraction:

11. $\frac{144}{529}$. **12.** $\frac{961}{1024}$. **13.** $\frac{1089}{1156}$. **14.** $\frac{1936}{5041}$.

Square Root with Decimals. Find the value of $\sqrt{151.29}$.

Show the class that the greatest square of the tens in 151.29 is 100, and that the square root of 100 is 10.

Then 51.29 contains $2 \times 10 \times$ the units' number of the root, plus the square of the units' number. Ask why this is the case.

Dividing 51 by 2×10, or 20, we find that the next figure of the root is 2.

We have now found 12, the square being $100 + 44 = 144$.

Then 7.29 contains $2 \times 12 \times$ the tenths' number of the root, plus the square of the tenths' number, because we have subtracted 144, which is the square of 12.

Dividing by 24, we find that the tenths' figure of the root is 3.

Hence the square root of 151.29 is 12.3.

If the number is not a perfect square, we may annex pairs of zeros at the right of the decimal point and find the root to as many decimal places as we choose.

	1 51.29 (12.3
	1
20	51
22	44
240	7 29
243	7 29

Summary of Square Root. We now see that the following are the steps to be taken in extracting square root:

Separate the number into periods of two figures each, beginning at the decimal point.

Find the greatest square in the left-hand period and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

Divide the new dividend thus obtained by twice the part of the root already found. Annex to this divisor the figure thus found and multiply by the number of this figure.

Subtract this result, bring down the next period, and proceed as before until all the periods have been thus annexed.

The result is the square root required.

Exercise 63. Square Root

Find the square roots of the following:

1. 12,321.	**5.** 19.4481.	**9.** 63,001.
2. 54,756.	**6.** 0.2809.	**10.** 21,224,449.
3. 110.25.	**7.** 1176.49.	**11.** 49,112,064.
4. 8046.09.	**8.** 82.2649.	**12.** 96,275,344.

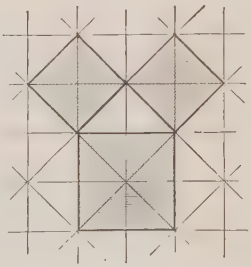
In Exs. 13–17 give the square roots to two decimal places only.

13. 2. **14.** 5. **15.** 7. **16.** 8. **17.** 11.

18. Find, to the nearest hundredth of an inch, the side of a square whose area is 3 sq. in.

Square on the Hypotenuse. As we learned on page 112, in a right triangle the side opposite the right angle is called the *hypotenuse.*

If a floor is made up of triangular tiles like this, it is easy to mark out a right triangle. In the figure it is seen that the square on the hypotenuse contains eight small triangles, while the square on each side contains four such triangles. Hence we see that



The square on the hypotenuse is equal to the sum of the squares on the other two sides.

This remarkable fact is proved in geometry for all right triangles.

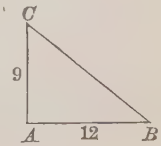
Given that $AB = 12$ and $AC = 9$, find BC.

Since $\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2$,

therefore $\overline{BC}^2 = 12^2 + 9^2$,

or $\overline{BC}^2 = 144 + 81 = 225$,

and $BC = \sqrt{225} = 15$.



Exercise 64. Square Root

1. How long is the diagonal of a floor 48 ft. by 75 ft.?

On this page state results to two decimal places only.

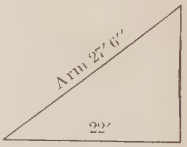
2. Find the length of the diagonal of a square that contains 9 sq. ft.

3. The two sides of a right triangle are 40 in. and 60 in. Find the length of the hypotenuse.

4. What is the length of a wire drawn taut from the top of a 75-foot building to a spot 40 ft. from the foot?

5. A telegraph pole is set perpendicular to the ground, and a taut wire, fastened to it 20 ft. above the ground, leads to a stake 15 ft. 6 in. from the foot of the pole, so as to hold it in place. How long is the wire?

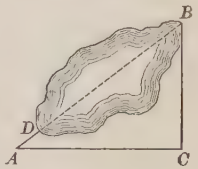
6. A derrick for hoisting coal has its arm 27 ft. 6 in. long. It swings over an opening 22 ft. from the base of the arm. How far is the top of the derrick above the opening?



Reversing the procedure on page 241, the square on either side is equal to the difference between what two squares?

7. The foot of a 45-foot ladder is 27 ft. from the wall of a building against which the top rests. How high does the ladder reach on the wall?

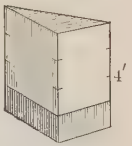
8. To find the length of this pond a class laid off the right triangle ACB as shown. It was found that $AC = 428$ ft., $BC = 321$ ft., and $AD = 75$ ft. Find DB.



9. How far from the wall of a house must the foot of a 36-foot ladder be placed so that the top may touch a window sill 32 ft. from the ground?

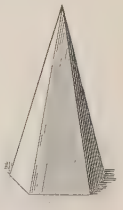
Prism. A solid in which the bases are equal polygons and the other faces are rectangles is called a *prism.*

Volume of a Prism. It is evident that we may find the volume of a prism in the same way that we found the volume of a cylinder (page 200). That is,

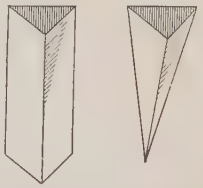


The volume of a prism is equal to the product of the base and height.

Pyramid. A solid of this shape in which the base is any polygon and the other faces are triangles meeting at a point is called a *pyramid.* The point at which the triangular faces meet is called the *vertex* of the pyramid, and the distance from the vertex to the base is called the *altitude* of the pyramid. The faces not including the base are called *lateral faces.*



Volume of a Pyramid. If we fill a hollow prism with water and then pour the water into a hollow pyramid of the same base and the same height, as here shown, it will be found that the pyramid has been filled exactly three times with the water that filled the prism. Therefore,



The volume of a pyramid is equal to one third the product of the base and height.

Lateral Surface of a Pyramid. The height of a lateral face of a pyramid is called the *slant height* of the pyramid.

Since the area of each lateral face of a pyramid is equal to half the product of the base and altitude,

The area of the lateral surface of a pyramid is equal to the perimeter of the base multiplied by half the slant height.

Lateral Surface of a Cone. If we should slit the surface of a cone and flatten it out, we would have part of a circle.

The terms "lateral surface" and "slant height" will be understood from the study of the pyramid.

From our study of the circle we infer that

The lateral surface of a cone is equal to the circumference of the base multiplied by half the slant height.

Volume of a Cone. In the way that we found the volume of a pyramid we may find the volume of a cone. Then

The volume of a cone is equal to one third the product of the base and height.

Find the volume of a cone of height 5 in. and radius 2 in.

Area of base is $\frac{22}{7} \times 4$ sq. in.

Volume is $\frac{1}{3} \times 5 \times \frac{22}{7} \times 4$ cu. in. $= 20.95$ cu. in.

This method of calculation gives $20.95\frac{5}{7}$ cu. in. as the volume, but 20.95 cu. in. is a much more practical form for the answer.

Teachers will observe that only the simplest forms of prisms, pyramids, and cones have been considered in this book.

Exercise 65. Lateral Surfaces and Volumes

Find the volumes of prisms and also the volumes of pyramids with the following bases and heights:

1. 36 sq. in., 7 in. **2.** 48 sq. in., $5\frac{1}{2}$ in. **3.** 5.7 sq. in., 4.8 in.

Find the lateral surfaces of pyramids with the following perimeters of bases and slant heights:

4. 30 in., 18 in. **5.** 3 ft. 3 in., 8 in. **6.** 5 ft. 9 in., 10 in.

Find the volumes of cones with the following radii of bases and heights:

7. 14 in., 6 in. **8.** 5.6 in., 15 in. **9.** 49 in., 15 in.

Surface of a Sphere. If we wind half of the surface of a sphere with a cord as here shown, and then wind with exactly the same length of the cord the surface of a

cylinder whose radius is equal to the radius of the sphere and whose height is equal to the diameter, we find that the cord covers half the curved surface of the cylinder.

Therefore the surface of a sphere is equal to the curved surface of a cylinder of the same radius and height.

We can now easily show that

$$\text{surface of sphere} = \tfrac{22}{7} \times 2 \times \text{radius} \times 2 \times \text{radius}.$$

Hence *the surface of a sphere is equal to 4 times* $\frac{22}{7}$ *times the square of the radius.*

Volume of a Sphere. It is shown in geometry that

The volume of a sphere is equal to $\frac{4}{3}$ *times* $\frac{22}{7}$ *times the cube of the radius.*

The cube of r means $r \times r \times r$ and is written r^3.

1. Considering the earth as a sphere of 4000 mi. radius, find the surface.

$$4 \times \tfrac{22}{7} \times 4000^2 = 4 \times \tfrac{22}{7} \times 16{,}000{,}000 = 201{,}142{,}857\tfrac{1}{7}.$$

Therefore the surface is about 201,143,000 sq. mi.

2. If a ball is 4 ft. in diameter, find the volume.

The radius is $\frac{1}{2}$ of 4 ft., or 2 ft.

The volume is $\frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2$ cu. ft., or 33.52 cu. ft.

Exercise 66. Surfaces and Volumes

1. If a ball has a radius of $1\frac{1}{2}$ in., find the surface.

2. Find the surface of a tennis ball of diameter $2\frac{3}{8}$ in.

3. If a cubic foot of granite weighs 165 lb., find the weight of a sphere of granite 4 ft. in diameter.

4. A bowl is in the form of a hemisphere 4.9 in. in diameter. How many cubic inches does it contain?

5. A ball 4′ 6″ in diameter for the top of a tower is to be gilded. How many square inches are to be gilded?

6. A pyramid has a lateral surface of 400 sq. in. The slant height is 16 in. Find the perimeter of the base.

7. A conic spire has a slant height of 34 ft. and the circumference of the base is 30 ft. Find the lateral surface.

8. What is the entire surface of a cone whose slant height is 6 ft. and the diameter of whose base is 6 ft.?

9. What is the weight of a sphere of marble 3 ft. in circumference, marble being 2.7 times as heavy as water and 1 cu. ft. of water weighing 1000 oz.?

10. Taking the earth as an exact sphere with radius 4000 mi., find the volume to the nearest 1000 cu. mi.

11. If 1 cu. ft. of a certain quality of marble weighs 173 lb., what is the weight of a cylindric marble column that is 12 ft. high and 18 in. in diameter?

12. How many cubic yards of earth must be removed in digging a canal 8 mi. 900 ft. long, 180 ft. wide, and 18 ft. deep?

13. A marble $1\frac{1}{2}$ in. in diameter is dropped into a cylindric jar 5 in. high and 4 in. in diameter, half full of water. How much does the marble cause the water to rise?

TABLES FOR REFERENCE

Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
$31\frac{1}{2}$ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)
365 days = 1 common year
366 days = 1 leap year

Value

10 mills = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)

Angles and Arcs

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)

Counting

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gr.)
12 gross, or 1728 units = 1 great gross

Paper

24 sheets = 1 quire
500 sheets = 1 ream

Formerly 480 sheets of paper were called a *ream*. The word "quire" is now used only for folded note paper, other paper being usually sold by the pound.

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